Latent Variable Methods Workshop
Longitudinal Data Analysis Models

Day 2 : Longitudinal data analysis with Mplus, R/lavaan

lvmworkshop.org

Brown University
Zoom September 3, 2025

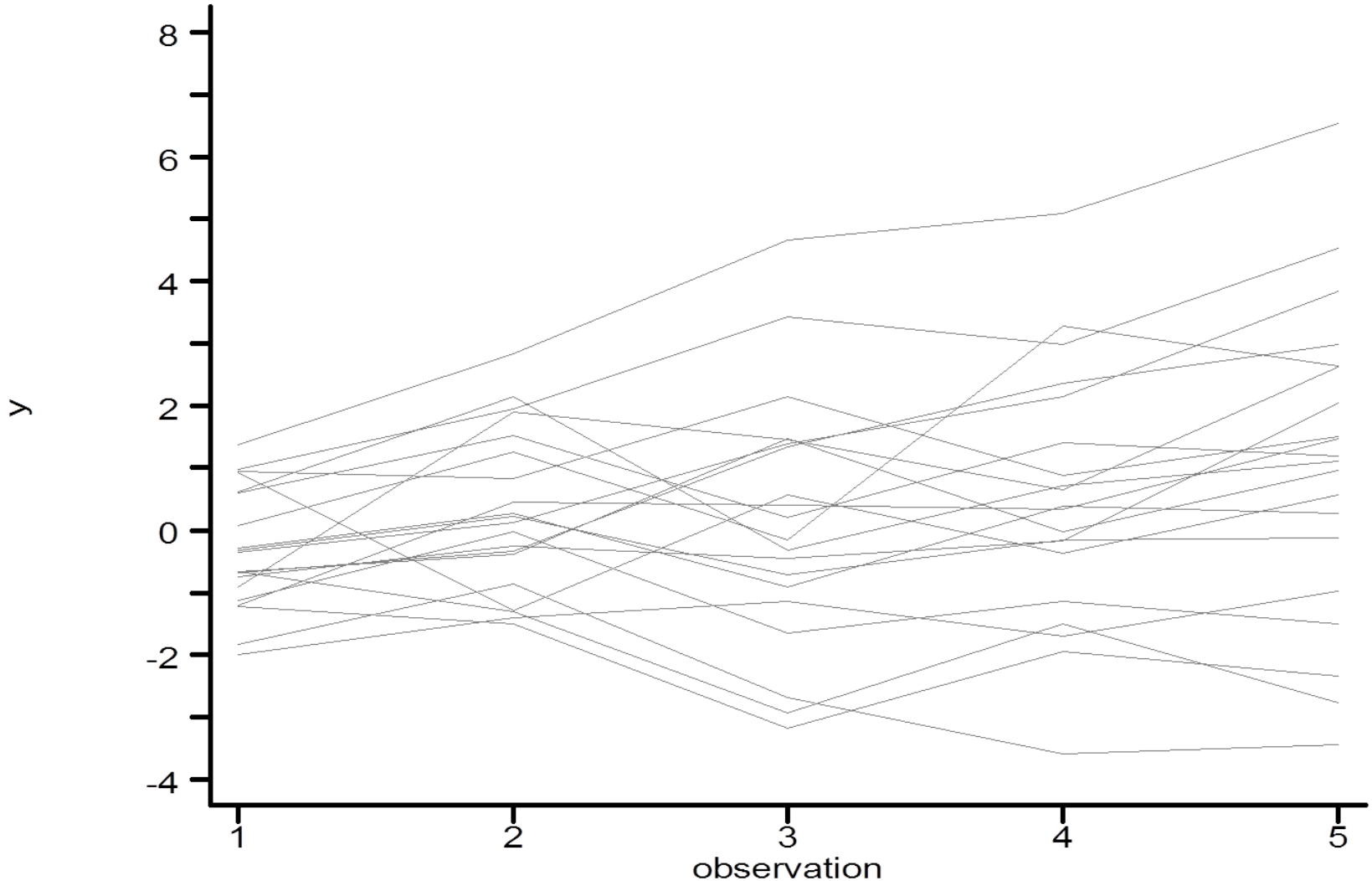
Session Overview

- Other Resources
- The Problem
- The Model
- Implementation in Mplus
- Example
- Extensions
- Discussion

Other Resources

- What is longitudinal data analysis?
  - Singer JD & Willett JB. Applied longitudinal data analysis: Modeling change and event occurrence. 2003, New York: Oxford University Press.
- How do I do latent growth curve modeling?
  - Duncan TE, Duncan SC, + Strycker LA. An introduction to latent variable growth curve modeling: concepts, issues and applications. Second ed. 2006, Mahwah, New Jersey: Lawrence Erlbaum Associates, Inc.
- Tell me more about the math behind latent curve methods
  - Bollen KA & Curran PJ. Latent curve models: a structural equation perspective. Wiley series in probability and statistics. 2006, Hoboken, N.J.: Wiley-Interscience.

The Problem



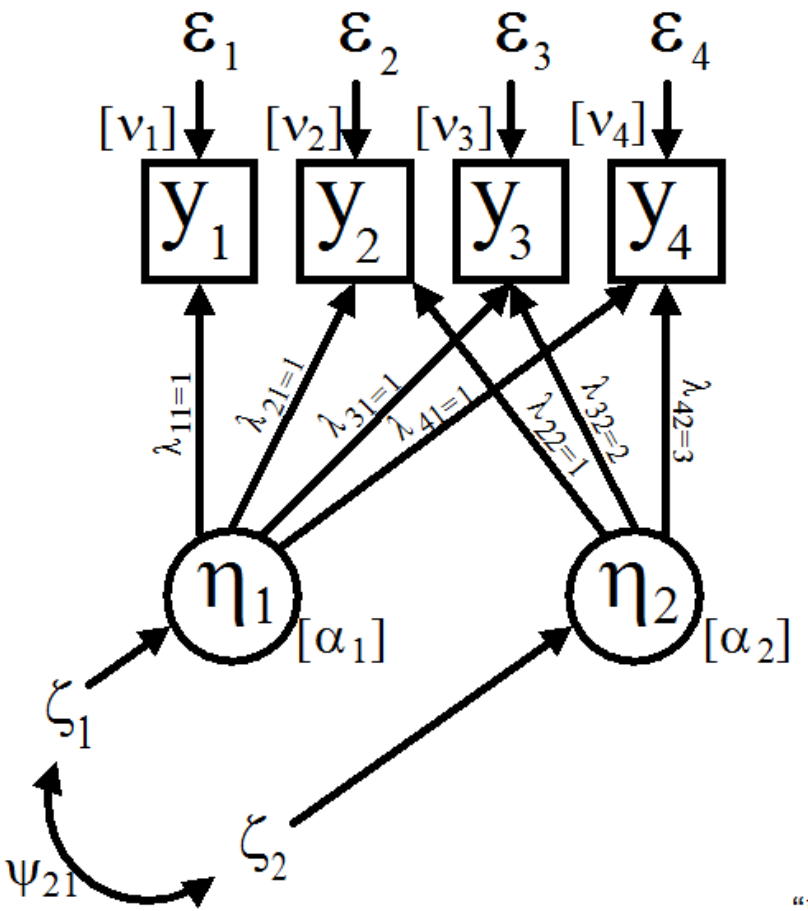
The Model

individual i at time t

$$
\begin{aligned}
y_{it} &= \nu_t + \eta_{1i}\lambda_{1t} + \eta_{2i}\lambda_{2ti} + \epsilon_{ti} \\
\eta_{1i} &= \alpha_1 + \zeta_{1i} \\
\eta_{2i} &= \alpha_2 + \zeta_{2i}
\end{aligned}
$$

A typical specification is $\nu = 0, \lambda_1 = 1$ for all t, and $\lambda_{2ti} = \lambda_{t2}$. An alternative specification is $\nu = \nu_1 = \nu_2 = \ldots = \nu_t \neq 0$, and $\alpha_1 = 0$. Many models (e.g. Proc Mixed) assume $V(\epsilon_t) = V(\epsilon)$

Latent Growth Model- Linear Change



Typical parameterization for linear change and equally-spaced time steps

$$\nu = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \qquad \Lambda = \begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 1 & 2 \\ 1 & 3 \end{bmatrix}$$

$$\theta = VAR(\varepsilon) = \begin{bmatrix} * & 0 & 0 & 0 \\ 0 & * & 0 & 0 \\ 0 & 0 & * & 0 \\ 0 & 0 & 0 & * \end{bmatrix}$$

$$\alpha = \begin{bmatrix} * \\ * \end{bmatrix}$$

$$\Psi = VAR(\zeta) = \begin{bmatrix} * & \\ * & * \end{bmatrix}$$

"*" Implies parameter freely estimated. All other parameters are held constant to the indicated value.

The Model

individual i at time t

$$
\begin{aligned}
y_{it} &= \eta_{1i}\lambda_{1t} + \eta_{2i}\lambda_{2t} + \epsilon_{ti} \\
\eta_{1i} &= \alpha_1 + \zeta_{1i} \\
\eta_{2i} &= \alpha_2 + \zeta_{2i}
\end{aligned}
$$

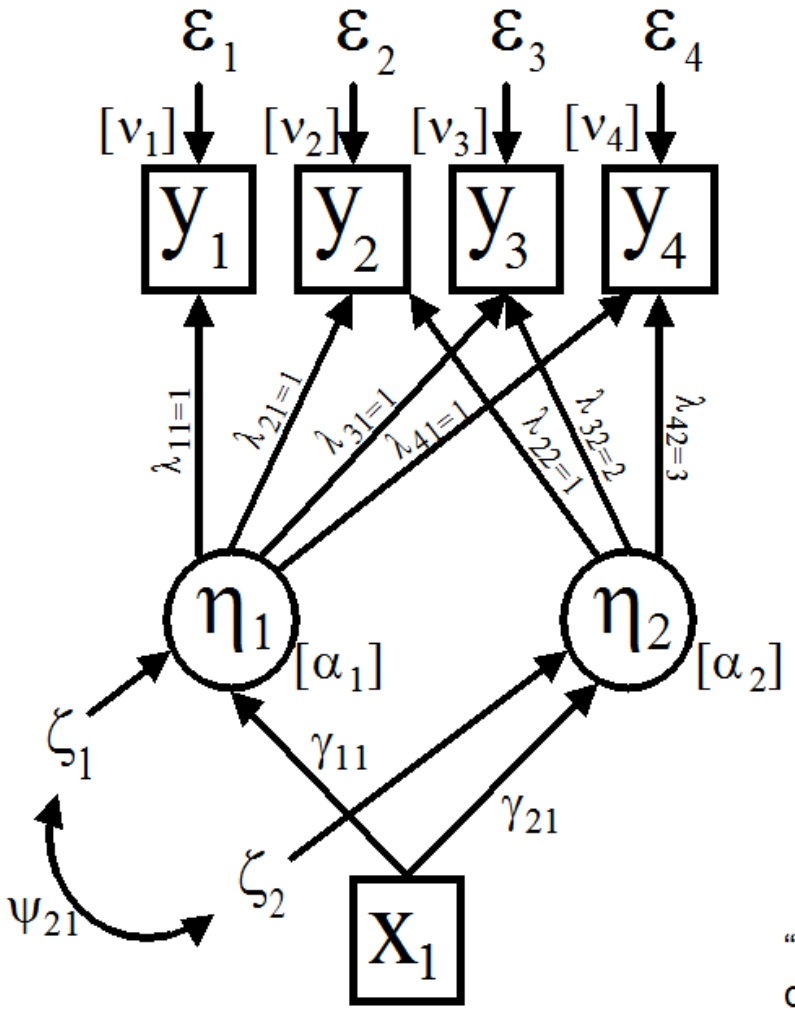
A typical specification for LGM

The Model

individual i at time t conditional on time invariant covariate x_{1i}

$$
\begin{aligned}
y_{it} &= \eta_{1i}\lambda_{1t} + \eta_{2i}\lambda_{2t} + \epsilon_{ti} \\
\eta_{1i} &= \alpha_1 + \gamma_{11}x_{1i} + \zeta_{1i} \\
\eta_{2i} &= \alpha_2 + \gamma_{21}x_{1i} + \zeta_{2i}
\end{aligned}
$$

Latent Growth Model- Linear Change



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$$\nu = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \qquad \Lambda = \begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 1 & 2 \\ 1 & 3 \end{bmatrix}$$

$$\theta = VAR(\varepsilon) = \begin{bmatrix} * & 0 & 0 & 0 \\ 0 & * & 0 & 0 \\ 0 & 0 & * & 0 \\ 0 & 0 & 0 & * \end{bmatrix}$$

$$\alpha = \begin{bmatrix} * \\ * \end{bmatrix} \qquad \Gamma = \begin{bmatrix} * \\ * \end{bmatrix}$$

$$\Psi = VAR(\zeta) = \begin{bmatrix} * & \\ * & * \end{bmatrix}$$

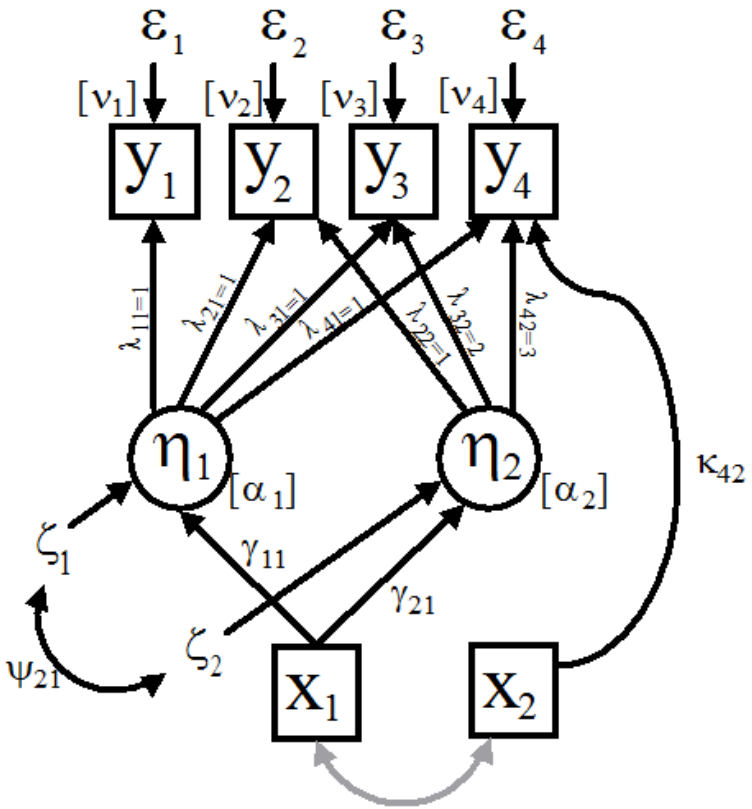
"*" Implies parameter freely estimated. All other parameters are held constant to the indicated value.

The Model

individual i at time t conditional on
time invariant covariate x_{1i}
and time-varying covariate x_{wi}

$$
\begin{aligned}
y_{it} &= \eta_{1i}\lambda_{1t} + \eta_{2i}\lambda_{2t} + k_i x_{wi} + \epsilon_{ti} \\
\eta_{1i} &= \alpha_1 + \gamma_{11} x_{1i} + \zeta_{1i} \\
\eta_{2i} &= \alpha_2 + \gamma_{21} x_{1i} + \zeta_{2i}
\end{aligned}
$$

Latent Growth Model- Linear Change



Typical parameterization for linear change and equally-spaced time steps

$$\nu = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \qquad \Lambda = \begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 1 & 2 \\ 1 & 3 \end{bmatrix}$$

$$\theta = VAR(\varepsilon) = \begin{bmatrix} * & 0 & 0 & 0 \\ 0 & * & 0 & 0 \\ 0 & 0 & * & 0 \\ 0 & 0 & 0 & * \end{bmatrix}$$

$$\alpha = \begin{bmatrix} * \\ * \end{bmatrix} \quad \Gamma = \begin{bmatrix} * \\ * \end{bmatrix} \quad \kappa = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & * \end{bmatrix}$$

$$\Psi = VAR(\zeta) = \begin{bmatrix} * & \\ * & * \end{bmatrix}$$

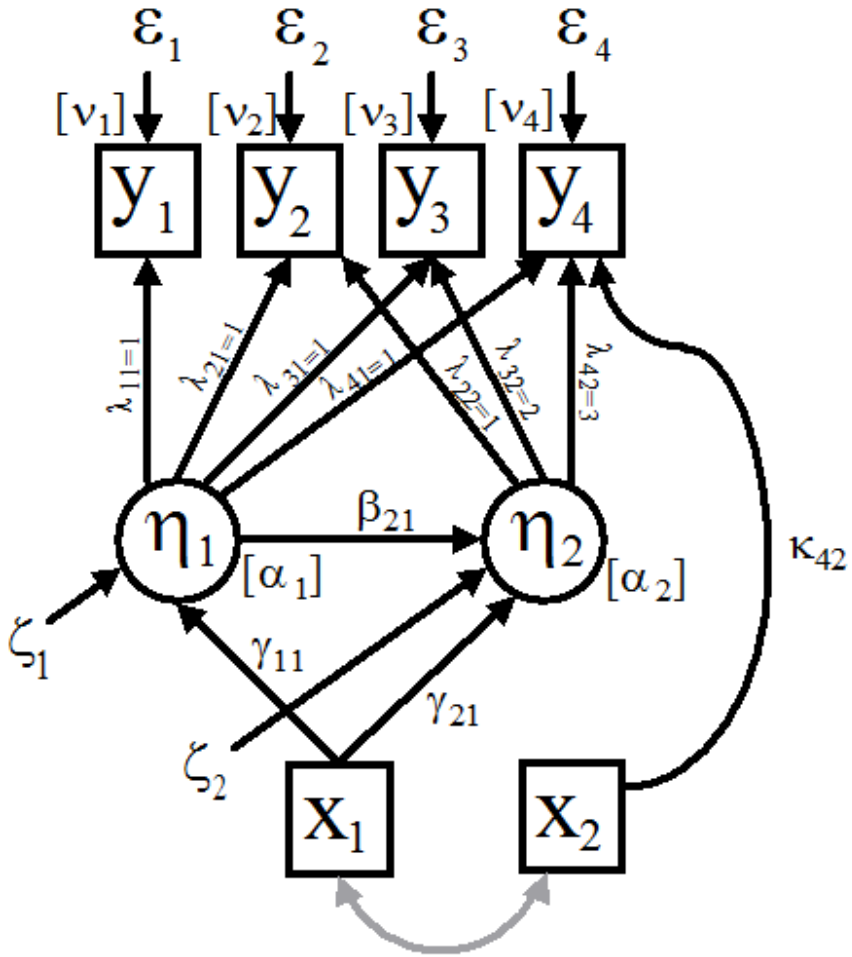
"*" Implies parameter freely estimated. All other parameters are held constant to the indicated value.

The Model

including a regression of change on baseline

$$
\begin{aligned}
y_{it} &= \eta_{1i}\lambda_{1t} + \eta_{2i}\lambda_{2t} + k_i x_{wi} + \epsilon_{ti} \\
\eta_{1i} &= \alpha_1 + \gamma_{11} x_{1i} + \zeta_{1i} \\
\eta_{2i} &= \alpha_2 + \gamma_{21} x_{1i} + \beta\eta_{1i} + \zeta_{2i}
\end{aligned}
$$

Latent Growth Model- Linear Change



Typical parameterization for linear change and equally-spaced time steps

$$\nu = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad \Lambda = \begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 1 & 2 \\ 1 & 3 \end{bmatrix} \quad \theta = VAR(\varepsilon) = \begin{bmatrix} * & 0 & 0 & 0 \\ 0 & * & 0 & 0 \\ 0 & 0 & * & 0 \\ 0 & 0 & 0 & * \end{bmatrix}$$

$$\alpha = \begin{bmatrix} * \\ * \end{bmatrix} \quad B = \begin{bmatrix} 0 & 0 \\ * & 0 \end{bmatrix} \quad \Psi = VAR(\zeta) = \begin{bmatrix} * & \\ * & * \end{bmatrix}$$

$$\Gamma = \begin{bmatrix} * \\ * \end{bmatrix} \quad \kappa = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & * \end{bmatrix}$$

"*" Implies parameter freely estimated. All other parameters are held constant to the indicated value.

Latent Growth Model

- Latent Growth Curve (LGC) modeling is just like CFA
- Reconceptualize "factors" as "random effects"
- Factor loadings are
  - (usually) not estimated but given by design or data, and
  - relate to the sequence of repeated observations
- The action is in the mean structure part of the model (factor means, item means, factor variances) as opposed to factor loadings

LGC Modeling In Mplus

- Mplus can accommodate
  - Individually varying times of observation
    - ⋆ Multivariate fashion with time scores
    - ⋆ Multiple record fashion with multilevel models
  - Random slopes among covariates

Model Building (LGC)

- Step 0: descriptive analysis, graphs
- Step 1: start with simple model (unconditional model, i.e. no covariates)
- Step 2: add covariates, regress intercept and slope on covariates.
- Step 3: Possible model modifications

Example

Question	How does mental status performance change over time among older adults?
Data Source	EPESE: Wave 1, 4, 7 Boston, New Haven, Iowa, Duke; N=14,456
Outcome	Mental Status Questionnaire (MSQ)
Covariates	Male, Black, Age

Step 0

Descriptive Analysis

Example Data Set

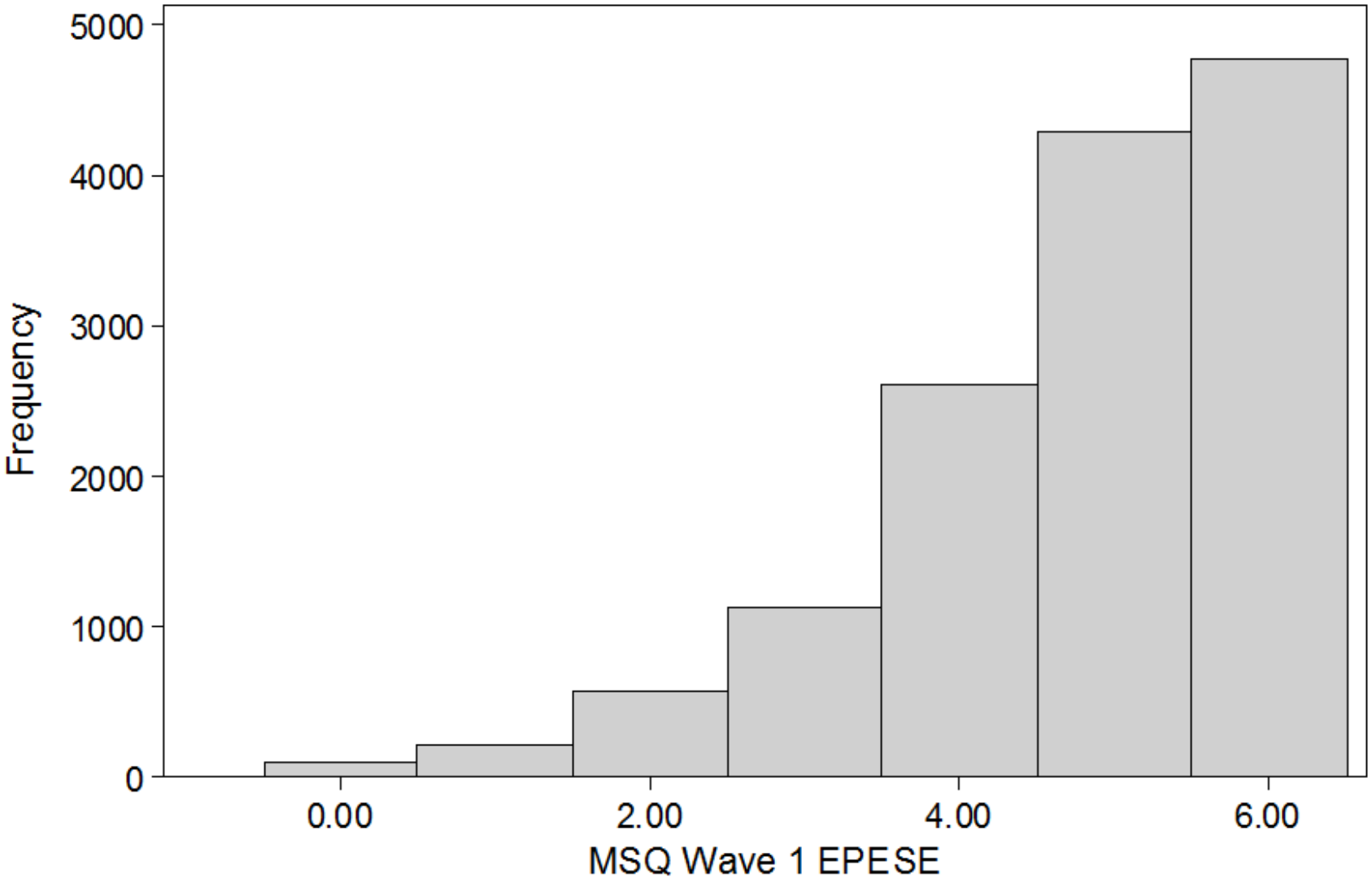
- EPESE
- N=14,456
- General population survey of older adults
- Carried out at four US sites:
  - East Boston, MA (N=3,809)
  - Iowa and Washington Counties, IA (N=3,673)
  - New Haven, CT(N=2,812)
  - Durham-Piedmont area of NC (N=4,162)
- Field work began 1981, 1985 in NC
- Data are in the public domain (ICPSR)
- Results we derive have not been published

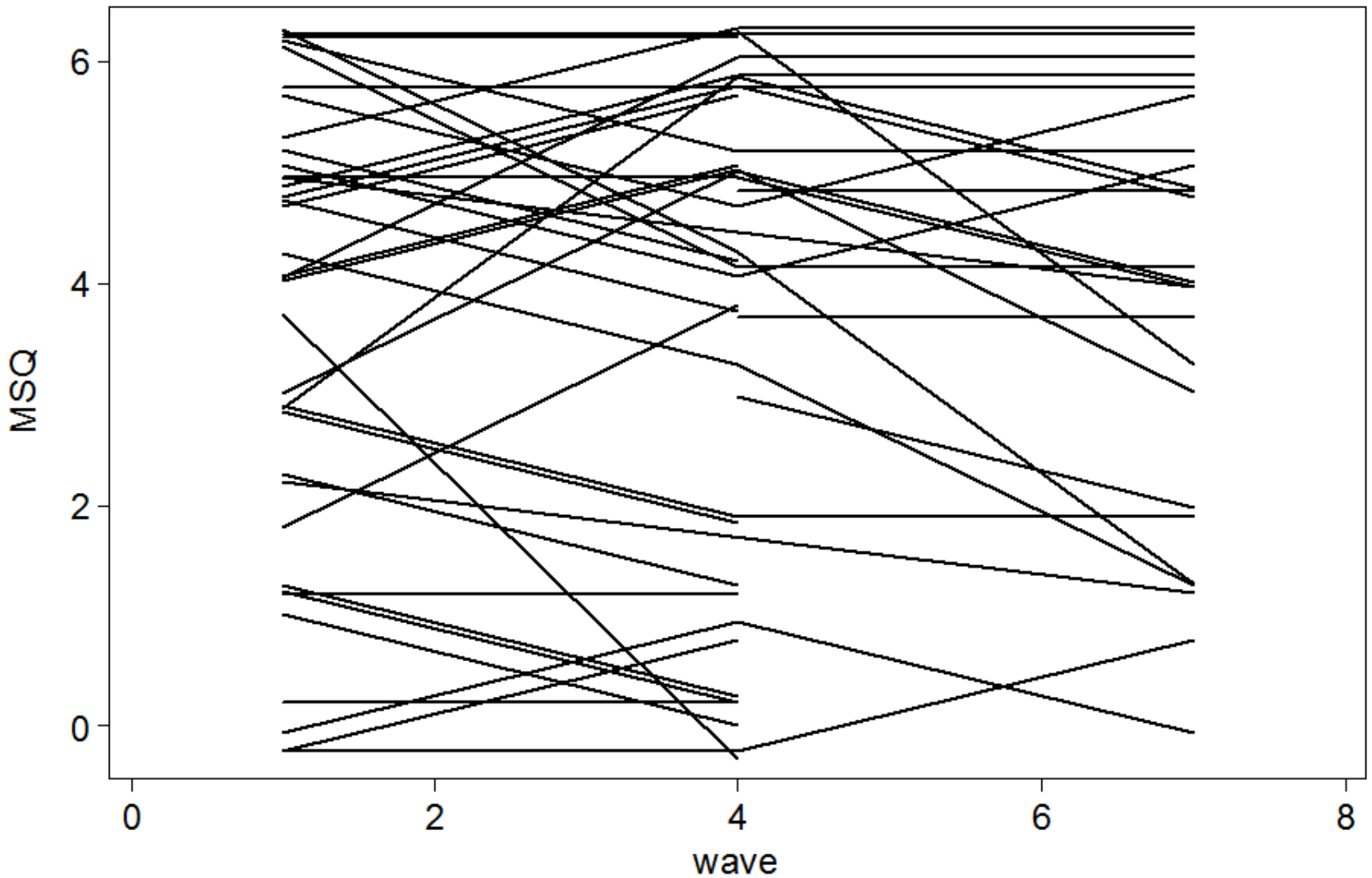
Mental Status Questionnaire

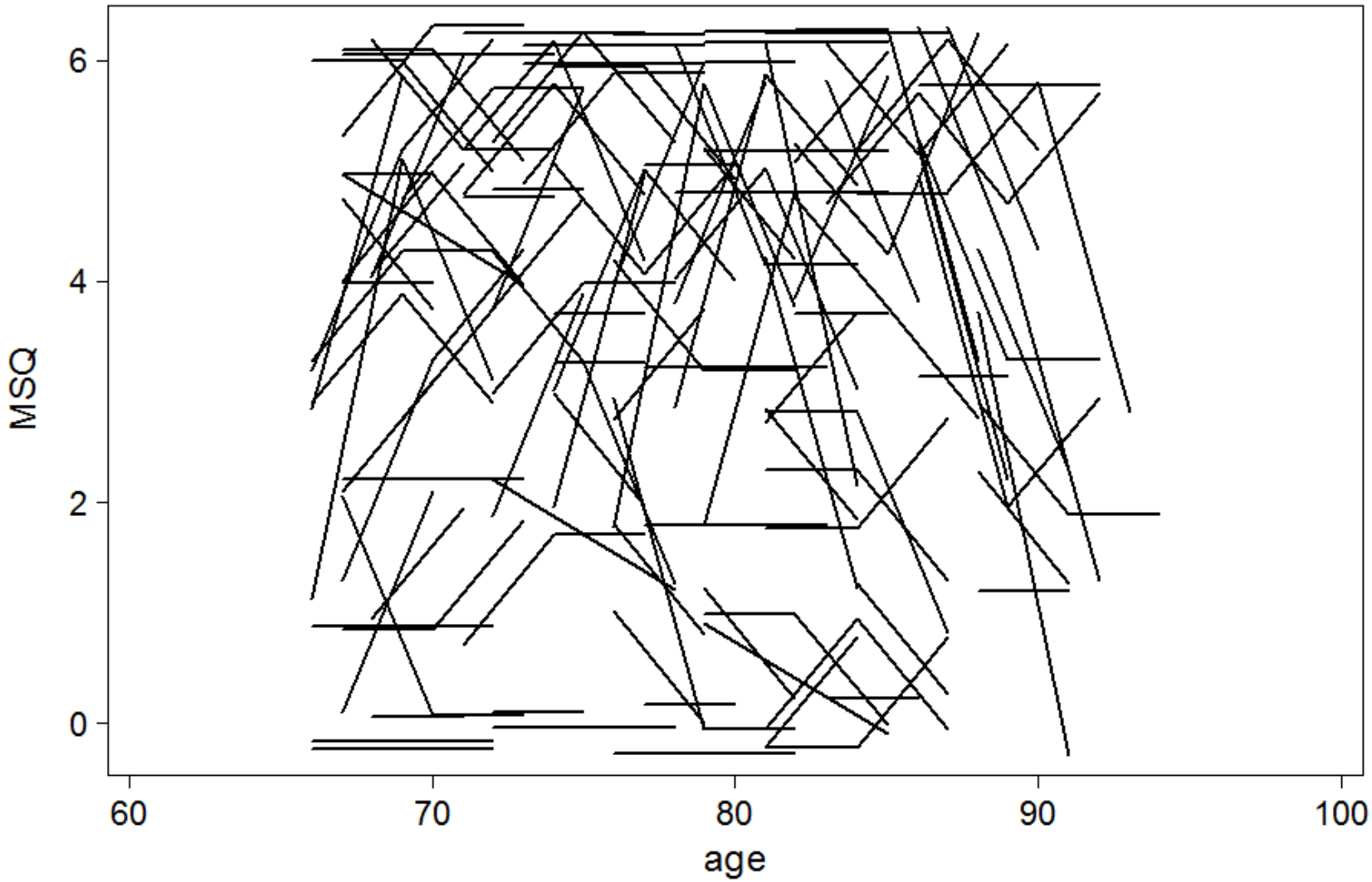
- Short Portable Mental Status Questionnaire
- We will use six items from EPESE
  - msq3 what is date
  - msq4 what is the day of week
  - msq5 who is the president
  - msq6 who was the prior president
  - msq7 what is your Mother's maiden name
  - msq9 Count back from 20 by 3's

MSQ Item / Response	MSQ item 3	4	5	6	7	9
1 correct	9,788	12,916	12,436	9,355	13,075	7,768
2 incorrect	3,907	772	1,239	4,302	539	5,194
9 refused	17	18	25	35	22	542
.	744	750	756	764	820	952
Total	14,456	14,456	14,456	14,456	14,456	14,456
1 correct	67.7	89.4	86.0	64.7	90.5	53.7
2 incorrect	27.0	5.3	8.6	29.8	3.7	35.9
9 refused	.1	.1	.2	.2	.2	3.7
.	5.1	5.2	5.2	5.3	5.7	6.6
Total	100.0	100.0	100.0	100.0	100.0	100.0

	date	day of week	Pres	Prior Pres	Mother Maiden	Count back 20 by 3


5000
4000
3000
2000
1000
0
Frequency
0.00
2.00
4.00
6.00
MSQ Wave 1 EPESE


MSQ
0
2
4
6
wave
0
2
4
6
8


MSQ
0
2
4
6
60
70
80
90
100
age

male	Freq.	Percent	Cum.
0	8,960	61.98	61.98
1	5,496	38.02	100.00
Total	14,456	100.00	

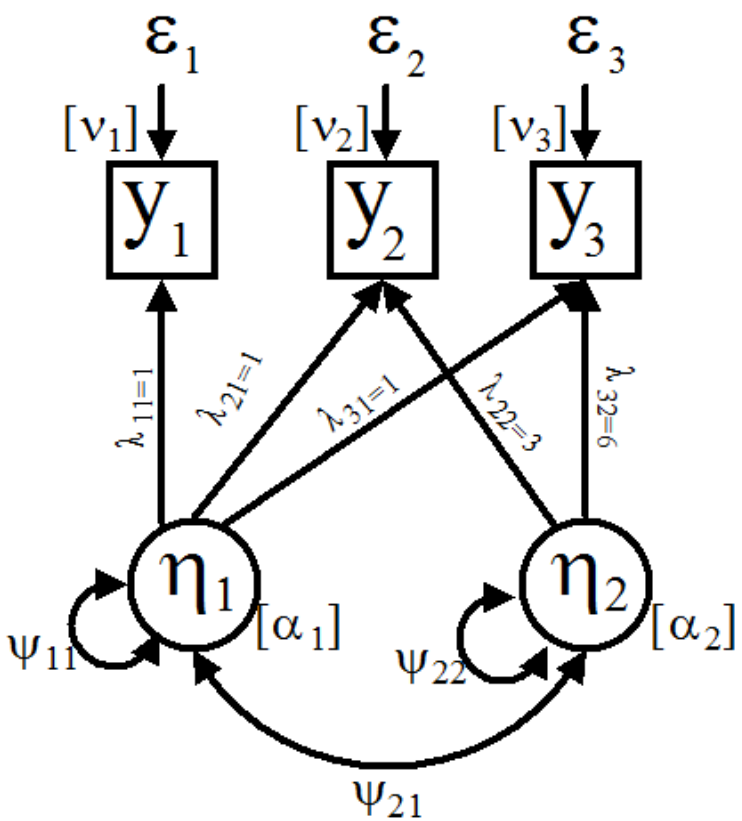
Black or African-American

site	0	1	.	Total
1 EB-MA	0	0	3,809	3,809
2 IW-IO	0	0	3,673	3,673
3 NH-CT	2,283	529	0	2,812
4 DP-NC	1,901	2,261	0	4,162
Total	4,184	2,790	7,482	14,456

age	Freq.	Percent	Cum.
67: 65-69	4,457	30.83	30.83
72: 70-74	3,970	27.46	58.29
77: 75-79	2,872	19.87	78.16
82: 80-84	1,878	12.99	91.15
87: 85+	1,279	8.85	100.00
Total	14,456	100.00	

Step 1

Start with a simple model
Time Basis: Time at assessment
Unconditional change in MSQ



Typical parameterization for linear change

$$\nu = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \qquad \Lambda = \begin{bmatrix} 1 & 0 \\ 1 & 3 \\ 1 & 6 \end{bmatrix}$$

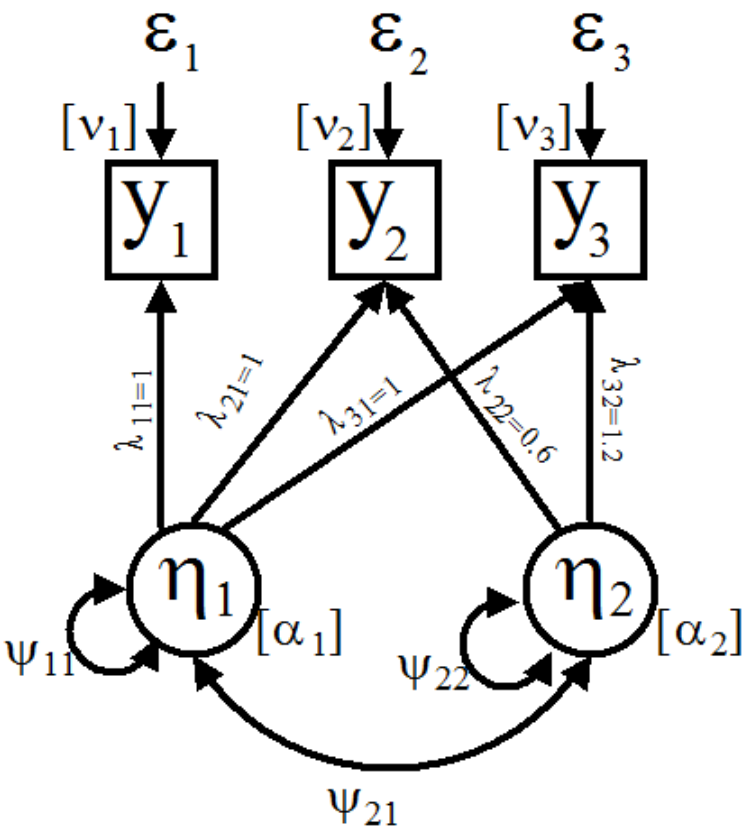
$$\theta = \begin{bmatrix} * & 0 & 0 \\ 0 & * & 0 \\ 0 & 0 & * \end{bmatrix}$$

$$\alpha = \begin{bmatrix} * \\ * \end{bmatrix} \qquad \Psi = \begin{bmatrix} * & \\ * & * \end{bmatrix}$$

"*" Implies parameter freely estimated. All other parameters are held constant to the indicated value.

Get Creative with Time

- In-person interviews at Baseline, Wave 4, Wave 7
  - Per-1-year effect, $\lambda = 0, 3, 6$
  - Per-5-year effect, $\lambda = 0, 0.6, 1.2$
  - Per-10-year effect, $\lambda = 0, 0.3, 0.6$
  - Total change over follow-up, $\lambda = 0, .5, 1$
  - General time basis, $\lambda = 0, *, c$ where c is any fixed constant
- Alternative specifications
  - Should not affect fit or precision of estimates, but
  - Will allow for interpretation of effects on a meaningful scale



Parameterization for linear change, where α_2 captures change over 5 years.

$$\nu = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \qquad \Lambda = \begin{bmatrix} 1 & 0 \\ 1 & 0.6 \\ 1 & 1.2 \end{bmatrix}$$

$$\theta = \begin{bmatrix} * & 0 & 0 \\ 0 & * & 0 \\ 0 & 0 & * \end{bmatrix}$$

$$\alpha = \begin{bmatrix} * \\ * \end{bmatrix} \qquad \Psi = \begin{bmatrix} * & \\ * & * \end{bmatrix}$$

"*" Implies parameter freely estimated. All other parameters are held constant to the indicated value.

ex0201.inp

```
TITLE:    Unconditional latent growth curve model
          MSQ sum scores from EPESE

DATA:     FILE = ex0201.dat;

VARIABLE: NAMES = msqtot1 msqtot4 msqtot7 ;
          MISSING  = ALL (-9999) ;

OUTPUT:   STANDARDIZED ;
          TECH1 ;

MODEL:    i s | msqtot1@0 msqtot4@.6 msqtot7@1.2;
```

```
Mplus VERSION 8.8 DEMO (Mac)
MUTHEN & MUTHEN
07/11/2022   1:40 PM

INPUT INSTRUCTIONS

  TITLE:    Unconditional latent growth curve model
            MSQ sum scores from EPESE

  DATA:     FILE = ex0201.dat;

  VARIABLE: NAMES = msqtot1 msqtot4 msqtot7 ;
            MISSING  = ALL (-9999) ;

  OUTPUT:   STANDARDIZED ;
            TECH1 ;

  MODEL:    i s | msqtot1@0 msqtot4@.6 msqtot7@1.2;




*** WARNING
  Data set contains cases with missing on all variables.
  These cases were not included in the analysis.
  Number of cases with missing on all variables:  397
   1 WARNING(S) FOUND IN THE INPUT INSTRUCTIONS


```

```
Unconditional latent growth curve model
MSQ sum scores from EPESE

SUMMARY OF ANALYSIS

Number of groups                                                 1
Number of observations                                       14059

Number of dependent variables                                    3
Number of independent variables                                  0
Number of continuous latent variables                            2

Observed dependent variables

  Continuous
   MSQTOT1     MSQTOT4     MSQTOT7

Continuous latent variables
   I           S


Estimator                                                       ML
Information matrix                                        OBSERVED
Maximum number of iterations                                 1000
Convergence criterion                                   0.500D-04
Maximum number of steepest descent iterations                  20
Maximum number of iterations for H1                          2000
Convergence criterion for H1                            0.100D-03

Input data file(s)
  ex0201.dat

```

```
Input data format  FREE


SUMMARY OF DATA

     Number of missing data patterns             7


COVARIANCE COVERAGE OF DATA

Minimum covariance coverage value   0.100


     PROPORTION OF DATA PRESENT


           Covariance Coverage
              MSQTOT1        MSQTOT4        MSQTOT7
              --------       --------       --------
 MSQTOT1        0.974
 MSQTOT4        0.752          0.775
 MSQTOT7        0.565          0.561          0.581


UNIVARIATE SAMPLE STATISTICS


     UNIVARIATE HIGHER-ORDER MOMENT DESCRIPTIVE STATISTICS

         Variable/          Mean/     Skewness/   Minimum/ % with
              Percentiles
```

```
 96          Sample Size       Variance    Kurtosis     Maximum   Min/Max       20%/60%     40%/
                  80%     Median
 97
 98       MSQTOT1                  4.768       -1.152       0.000     0.70%        4.000
               5.000       5.000
 99              13698.000         1.621        1.156       6.000    34.84%        5.000
                     6.000
100       MSQTOT4                  4.732       -1.233       0.000     1.17%        4.000
               5.000       5.000
101             10899.000         1.822        1.268       6.000    35.05%        5.000
                     6.000
102       MSQTOT7                  4.569       -1.160       0.000     2.00%        4.000
               5.000       5.000
103               8169.000         2.033        1.060       6.000    30.31%        5.000
                     6.000
104
105
106 THE MODEL ESTIMATION TERMINATED NORMALLY
107
108
109
110 MODEL FIT INFORMATION
111
112 Number of Free Parameters                        8
113
114 Loglikelihood
115
116           H0 Value                      -52691.657
117           H1 Value                      -52661.224
118
119 Information Criteria
120
```

```
121           Akaike (AIC)                    105399.315
122           Bayesian (BIC)                  105459.723
123           Sample-Size Adjusted BIC        105434.300
124             (n* = (n + 2) / 24)
125
126 Chi-Square Test of Model Fit
127
128           Value                              60.866
129           Degrees of Freedom                     1
130           P-Value                            0.0000
131
132 RMSEA (Root Mean Square Error Of Approximation)
133
134           Estimate                           0.065
135           90 Percent C.I.                    0.052  0.080
136           Probability RMSEA <= .05           0.031
137
138 CFI/TLI
139
140           CFI                                0.991
141           TLI                                0.973
142
143 Chi-Square Test of Model Fit for the Baseline Model
144
145           Value                           6609.379
146           Degrees of Freedom                     3
147           P-Value                          0.0000
148
149 SRMR (Standardized Root Mean Square Residual)
150
151           Value                              0.024
152
```

```
153
154
155 MODEL RESULTS
156
157                                                    Two-Tailed
158                     Estimate       S.E.  Est./S.E.    P-Value
159
160  I        |
161     MSQTOT1            1.000      0.000    999.000    999.000
162     MSQTOT4            1.000      0.000    999.000    999.000
163     MSQTOT7            1.000      0.000    999.000    999.000
164
165  S        |
166     MSQTOT1            0.000      0.000    999.000    999.000
167     MSQTOT4            0.600      0.000    999.000    999.000
168     MSQTOT7            1.200      0.000    999.000    999.000
169
170  S        WITH
171     I                 -0.013      0.035     -0.385      0.700
172
173  Means
174     I                  4.785      0.011    448.325      0.000
175     S                 -0.309      0.012    -25.036      0.000
176
177  Intercepts
178     MSQTOT1            0.000      0.000    999.000    999.000
179     MSQTOT4            0.000      0.000    999.000    999.000
180     MSQTOT7            0.000      0.000    999.000    999.000
181
182  Variances
183     I                  1.002      0.033     30.688      0.000
184     S                  0.435      0.050      8.777      0.000
```

```
185
186  Residual Variances
187     MSQTOT1            0.620      0.031     19.805      0.000
188     MSQTOT4            0.807      0.018     45.489      0.000
189     MSQTOT7            0.692      0.038     18.411      0.000
190
191
192 QUALITY OF NUMERICAL RESULTS
193
194      Condition Number for the Information Matrix              0.301E-02
195        (ratio of smallest to largest eigenvalue)
196
197
198 STANDARDIZED MODEL RESULTS
199
200
201 STDYX Standardization
202
203                                                        Two-Tailed
204                     Estimate       S.E.  Est./S.E.    P-Value
205
206  I        |
207     MSQTOT1            0.786      0.012     65.942      0.000
208     MSQTOT4            0.717      0.010     74.818      0.000
209     MSQTOT7            0.662      0.013     52.207      0.000
210
211  S        |
212     MSQTOT1            0.000      0.000    999.000    999.000
213     MSQTOT4            0.283      0.015     18.393      0.000
214     MSQTOT7            0.523      0.031     17.045      0.000
215
216  S        WITH
```

```
    I                 -0.020       0.051      -0.395       0.693

 Means
    I                  4.781       0.079      60.151       0.000
    S                 -0.468       0.033     -14.150       0.000

 Intercepts
    MSQTOT1            0.000       0.000     999.000     999.000
    MSQTOT4            0.000       0.000     999.000     999.000
    MSQTOT7            0.000       0.000     999.000     999.000

 Variances
    I                  1.000       0.000     999.000     999.000
    S                  1.000       0.000     999.000     999.000

 Residual Variances
    MSQTOT1            0.382       0.019      20.423       0.000
    MSQTOT4            0.414       0.007      58.896       0.000
    MSQTOT7            0.302       0.016      19.375       0.000


STDY Standardization

                                                        Two-Tailed
                    Estimate       S.E.  Est./S.E.    P-Value

 I        |
    MSQTOT1            0.786       0.012      65.942       0.000
    MSQTOT4            0.717       0.010      74.818       0.000
    MSQTOT7            0.662       0.013      52.207       0.000

 S        |
```

```
    MSQTOT1            0.000      0.000    999.000    999.000
    MSQTOT4            0.283      0.015     18.393      0.000
    MSQTOT7            0.523      0.031     17.045      0.000

S        WITH
    I                 -0.020      0.051     -0.395      0.693

 Means
    I                  4.781      0.079     60.151      0.000
    S                 -0.468      0.033    -14.150      0.000

 Intercepts
    MSQTOT1            0.000      0.000    999.000    999.000
    MSQTOT4            0.000      0.000    999.000    999.000
    MSQTOT7            0.000      0.000    999.000    999.000

 Variances
    I                  1.000      0.000    999.000    999.000
    S                  1.000      0.000    999.000    999.000

 Residual Variances
    MSQTOT1            0.382      0.019     20.423      0.000
    MSQTOT4            0.414      0.007     58.896      0.000
    MSQTOT7            0.302      0.016     19.375      0.000


STD Standardization

                                                    Two-Tailed
                    Estimate       S.E.  Est./S.E.    P-Value

I        |
```

```
281     MSQTOT1            1.001      0.016     61.376      0.000
282     MSQTOT4            1.001      0.016     61.376      0.000
283     MSQTOT7            1.001      0.016     61.376      0.000
284
285  S        |
286     MSQTOT1            0.000      0.000    999.000    999.000
287     MSQTOT4            0.396      0.023     17.554      0.000
288     MSQTOT7            0.791      0.045     17.554      0.000
289
290  S        WITH
291     I                 -0.020      0.051     -0.395      0.693
292
293  Means
294     I                  4.781      0.079     60.151      0.000
295     S                 -0.468      0.033    -14.150      0.000
296
297  Intercepts
298     MSQTOT1            0.000      0.000    999.000    999.000
299     MSQTOT4            0.000      0.000    999.000    999.000
300     MSQTOT7            0.000      0.000    999.000    999.000
301
302  Variances
303     I                  1.000      0.000    999.000    999.000
304     S                  1.000      0.000    999.000    999.000
305
306  Residual Variances
307     MSQTOT1            0.620      0.031     19.805      0.000
308     MSQTOT4            0.807      0.018     45.489      0.000
309     MSQTOT7            0.692      0.038     18.411      0.000
310
311
312 R-SQUARE
```

```
313
314     Observed                                                Two-Tailed
315     Variable         Estimate        S.E.  Est./S.E.    P-Value
316
317     MSQTOT1             0.618       0.019     32.971       0.000
318     MSQTOT4             0.586       0.007     83.323       0.000
319     MSQTOT7             0.698       0.016     44.697       0.000
320
321
322 TECHNICAL 1 OUTPUT
323
324
325      PARAMETER SPECIFICATION
326
327
328             NU
329                MSQTOT1       MSQTOT4       MSQTOT7
330                --------      --------      --------
331                    0             0             0
332
333
334             LAMBDA
335                I             S
336                --------      --------
337  MSQTOT1             0             0
338  MSQTOT4             0             0
339  MSQTOT7             0             0
340
341
342             THETA
343                MSQTOT1       MSQTOT4       MSQTOT7
344                --------      --------      --------
```

```
MSQTOT1             1
MSQTOT4             0             2
MSQTOT7             0             0             3


           ALPHA
              I             S
              --------      --------
                  4             5


           BETA
              I             S
              --------      --------
I                   0             0
S                   0             0


           PSI
              I             S
              --------      --------
I                   6
S                   7             8


     STARTING VALUES


           NU
              MSQTOT1       MSQTOT4       MSQTOT7
              --------      --------      --------
                0.000         0.000         0.000
```

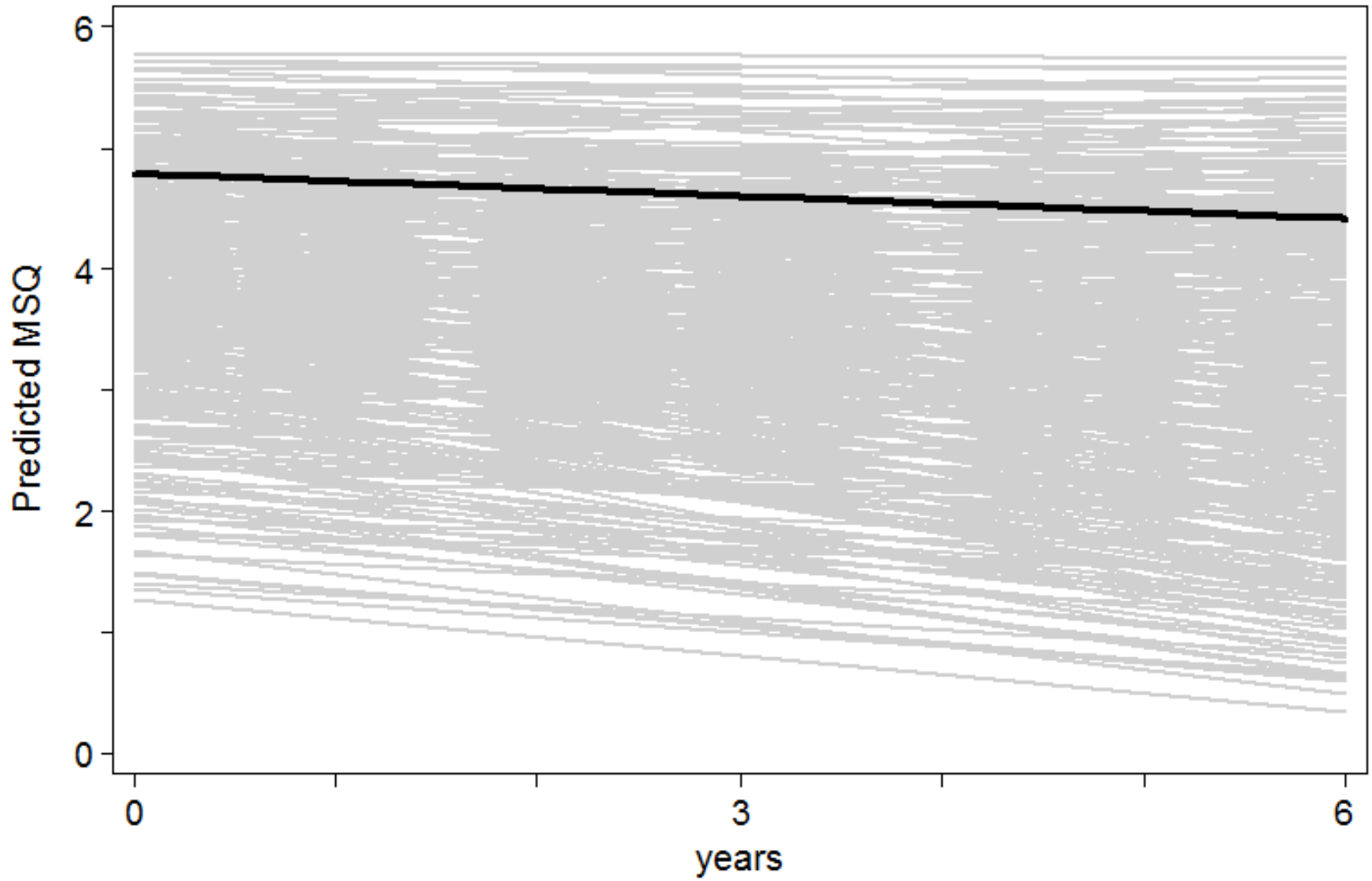
```
377
378
379             LAMBDA
380                I             S
381                --------      --------
382  MSQTOT1         1.000         0.000
383  MSQTOT4         1.000         0.600
384  MSQTOT7         1.000         1.200
385
386
387             THETA
388                MSQTOT1       MSQTOT4       MSQTOT7
389                --------      --------      --------
390  MSQTOT1         0.810
391  MSQTOT4         0.000         0.911
392  MSQTOT7         0.000         0.000         1.016
393
394
395             ALPHA
396                I             S
397                --------      --------
398                  4.948        -0.406
399
400
401             BETA
402                I             S
403                --------      --------
404  I               0.000         0.000
405  S               0.000         0.000
406
407
408             PSI
```

```
409                I              S
410                --------       --------
411  I               1.247
412  S               0.000          2.157
413
414
415       Beginning Time:  13:40:45
416          Ending Time:  13:40:45
417        Elapsed Time:  00:00:00
418
419
420 Mplus VERSION 8.8 DEMO (Mac) has the following limitations:
421   Maximum number of dependent variables: 6
422   Maximum number of independent variables: 2
423   Maximum number of between variables: 2
424   Maximum number of continuous latent variables in time series analysis: 2
425
426
427 MUTHEN & MUTHEN
428 3463 Stoner Ave.
429 Los Angeles, CA  90066
430
431 Tel: (310) 391-9971
432 Fax: (310) 391-8971
433 Web: www.StatModel.com
434 Support: Support@StatModel.com
435
436 Copyright (c) 1998-2022 Muthen & Muthen
```

ex0201.out

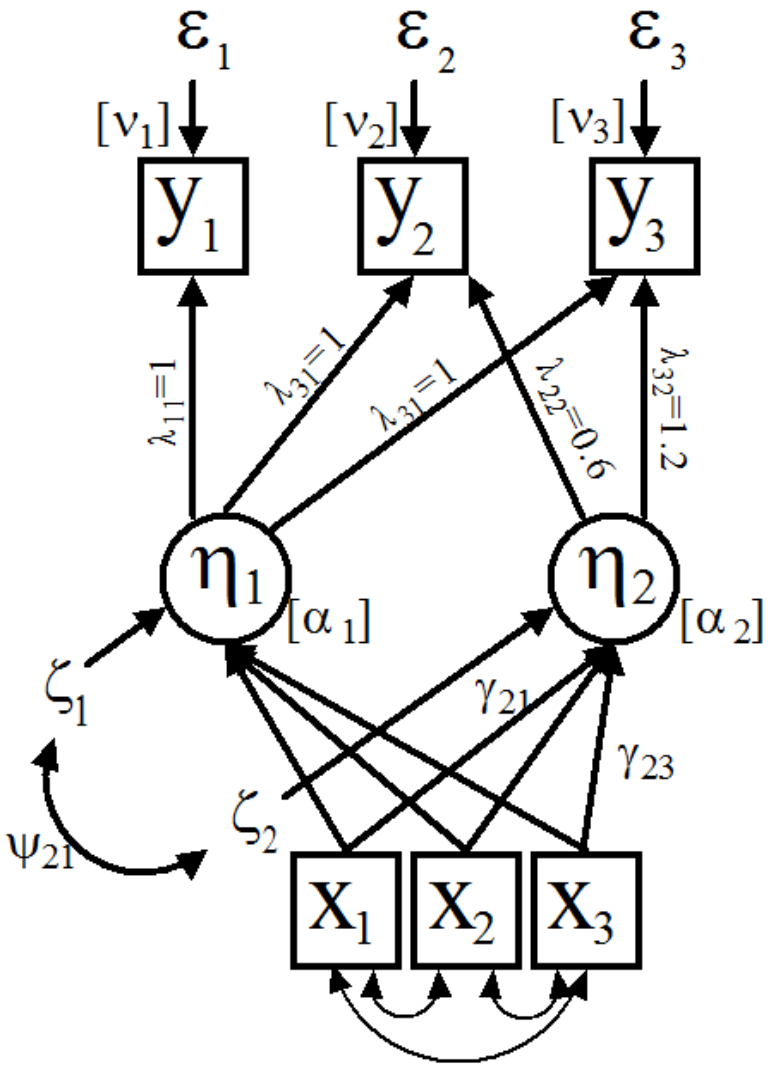
Post-Estimation Fit Evaluation

- Save factor scores
- Import into general purpose stat package
- Compute expected scores & residuals
- Graph them
- Empirical r^2
  - Correlation of true MSQTOT with model-implied (given i, s).
  - In this example is $r = 0.91$, so
  - $r^2 = 0.82$


Predicted MSQ
6
4
2
0
0
3
6
years

Step 2

Accept a well-fitting unconditional model,
and add covariates



Typical parameterization for linear change and equally-spaced time steps

$$\nu = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \qquad \Lambda = \begin{bmatrix} 1 & 0 \\ 1 & 0.6 \\ 1 & 1.2 \end{bmatrix}$$

$$\theta = VAR(\varepsilon) = \begin{bmatrix} * & 0 & 0 \\ 0 & * & 0 \\ 0 & 0 & * \end{bmatrix}$$

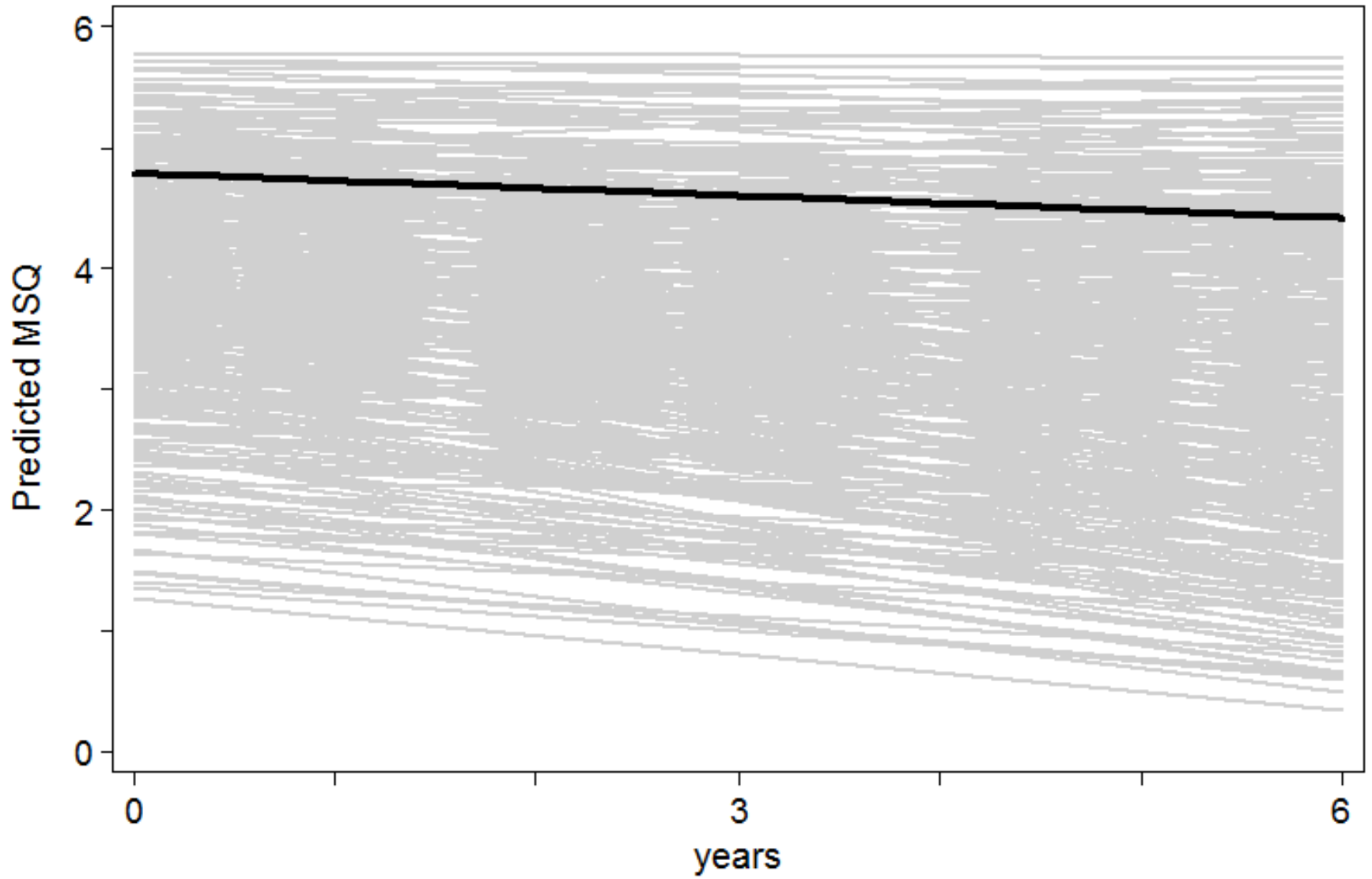
$$\alpha = \begin{bmatrix} * \\ * \end{bmatrix} \qquad \Gamma = \begin{bmatrix} * & * & * \\ * & * & * \end{bmatrix}$$

$$\Psi = \begin{bmatrix} * & \\ * & * \end{bmatrix}$$

"*" Implies parameter freely estimated. All other parameters are held constant to the indicated value.

Centering Predictors

- A powerful technique for all your regression activities
- See Kraemer & Blasey (2004) Int J Methods Psychatr Res 13(3):141-51
- Place intercept parameters on meaningful scale, mean values for the sample, by mean centering
- Same fit, same slopes, different intercepts
- Intercepts from conditional model will match unconditional model
- Mplus will center variables.
  - Version 7 syntax is different from earlier versions


Predicted MSQ
6
4
2
0
0
3
6
years

ex0202.inp

```
TITLE:    Conditional latent growth curve model
          MSQ sum scores from EPESE

DATA:     FILE = ex0201.dat;

VARIABLE: NAMES = msqtot1 msqtot4 msqtot7 age black ;
          MISSING  = ALL (-9999) ;

DEFINE:   CENTER age  (GRANDMEAN) ;

OUTPUT:   STANDARDIZED ;
          TECH1 ;

MODEL:    i s | msqtot1@0 msqtot4@.6 msqtot7@1.2;
          i on age      ; !@0 ;
          i on black    ; !@0 ;
          s on age      ; !@0 ;
          s on black    ; !@0 ;
```

```
Mplus VERSION 8.8 DEMO (Mac)
MUTHEN & MUTHEN
07/11/2022   1:40 PM

INPUT INSTRUCTIONS

  TITLE:    Conditional latent growth curve model
            MSQ sum scores from EPESE

  DATA:     FILE = ex0201.dat;

  VARIABLE: NAMES = msqtot1 msqtot4 msqtot7 age black ;
            MISSING  = ALL (-9999) ;

  DEFINE:   CENTER age  (GRANDMEAN) ;

  OUTPUT:   STANDARDIZED ;
            TECH1 ;

  MODEL:    i s | msqtot1@0 msqtot4@.6 msqtot7@1.2;
            i on age      ; !@0 ;
            i on black    ; !@0 ;
            s on age      ; !@0 ;
            s on black    ; !@0 ;
```

```
33
34
35 *** WARNING
36   Data set contains cases with missing on all variables except
37   x-variables.  These cases were not included in the analysis.
38   Number of cases with missing on all variables except x-variables:  397
39    1 WARNING(S) FOUND IN THE INPUT INSTRUCTIONS
40
41
42
43 Conditional latent growth curve model
44 MSQ sum scores from EPESE
45
46 SUMMARY OF ANALYSIS
47
48 Number of groups                                                 1
49 Number of observations                                       14059
50
51 Number of dependent variables                                  3
52 Number of independent variables                                2
53 Number of continuous latent variables                          2
54
55 Observed dependent variables
56
57   Continuous
58    MSQTOT1     MSQTOT4     MSQTOT7
59
60 Observed independent variables
61    AGE         BLACK
62
63 Continuous latent variables
64    I           S
```

```
65
66 Variables with special functions
67
68   Centering (GRANDMEAN)
69    AGE
70
71
72 Estimator                                                ML
73 Information matrix                                 OBSERVED
74 Maximum number of iterations                           1000
75 Convergence criterion                             0.500D-04
76 Maximum number of steepest descent iterations            20
77 Maximum number of iterations for H1                    2000
78 Convergence criterion for H1                      0.100D-03
79
80 Input data file(s)
81   ex0201.dat
82
83 Input data format  FREE
84
85
86 SUMMARY OF DATA
87
88      Number of missing data patterns             7
89
90
91 COVARIANCE COVERAGE OF DATA
92
93 Minimum covariance coverage value   0.100
94
95
96      PROPORTION OF DATA PRESENT
```

```
 97
 98
 99            Covariance Coverage
100               MSQTOT1        MSQTOT4        MSQTOT7        AGE            BLACK
101               --------       --------       --------       --------       --------
102  MSQTOT1        0.974
103  MSQTOT4        0.752          0.775
104  MSQTOT7        0.565          0.561          0.581
105  AGE            0.974          0.775          0.581          1.000
106  BLACK          0.974          0.775          0.581          1.000          1.000
107
108
109
110 UNIVARIATE SAMPLE STATISTICS
111
112
113      UNIVARIATE HIGHER-ORDER MOMENT DESCRIPTIVE STATISTICS
114
115          Variable/          Mean/     Skewness/   Minimum/ % with
                 Percentiles
116         Sample Size      Variance    Kurtosis    Maximum  Min/Max      20%/60%    40%/
                80%    Median
117
118      MSQTOT1               4.768       -1.152       0.000    0.70%       4.000
             5.000       5.000
119            13698.000       1.621        1.156       6.000   34.84%       5.000
                   6.000
120      MSQTOT4               4.732       -1.233       0.000    1.17%       4.000
             5.000       5.000
121            10899.000       1.822        1.268       6.000   35.05%       5.000
                   6.000
```

```
122      MSQTOT7                4.569       -1.160        0.000     2.00%        4.000
              5.000       5.000
123             8169.000        2.033        1.060        6.000    30.31%        5.000
                    6.000
124      AGE                    0.000        0.583       -6.932    31.36%       -6.932
              -1.932      -1.932
125             14059.000       40.275       -0.736       13.068     8.17%        3.068
                    8.068
126      BLACK                  0.196        1.528        0.000    80.35%        0.000
              0.000       0.000
127             14059.000        0.158        0.335        1.000    19.65%        0.000
                    0.000
128
129
130 THE MODEL ESTIMATION TERMINATED NORMALLY
131
132
133
134 MODEL FIT INFORMATION
135
136 Number of Free Parameters                        12
137
138 Loglikelihood
139
140           H0 Value                        -51447.318
141           H1 Value                        -51405.248
142
143 Information Criteria
144
145           Akaike (AIC)                    102918.636
146           Bayesian (BIC)                  103009.248
147           Sample-Size Adjusted BIC        102971.113
```

```
148             (n* = (n + 2) / 24)
149
150 Chi-Square Test of Model Fit
151
152           Value                             84.140
153           Degrees of Freedom                    3
154           P-Value                           0.0000
155
156 RMSEA (Root Mean Square Error Of Approximation)
157
158           Estimate                           0.044
159           90 Percent C.I.                    0.036  0.052
160           Probability RMSEA <= .05           0.885
161
162 CFI/TLI
163
164           CFI                                0.991
165           TLI                                0.973
166
167 Chi-Square Test of Model Fit for the Baseline Model
168
169           Value                          9121.331
170           Degrees of Freedom                    9
171           P-Value                           0.0000
172
173 SRMR (Standardized Root Mean Square Residual)
174
175           Value                              0.022
176
177
178
179 MODEL RESULTS
```

```
180
181                                                    Two-Tailed
182                     Estimate       S.E.  Est./S.E.    P-Value
183
184  I        |
185     MSQTOT1            1.000      0.000    999.000    999.000
186     MSQTOT4            1.000      0.000    999.000    999.000
187     MSQTOT7            1.000      0.000    999.000    999.000
188
189  S        |
190     MSQTOT1            0.000      0.000    999.000    999.000
191     MSQTOT4            0.600      0.000    999.000    999.000
192     MSQTOT7            1.200      0.000    999.000    999.000
193
194  I        ON
195     AGE               -0.046      0.002    -29.205      0.000
196     BLACK             -0.708      0.025    -28.044      0.000
197
198  S        ON
199     AGE               -0.032      0.002    -15.724      0.000
200     BLACK             -0.108      0.029     -3.683      0.000
201
202  S        WITH
203     I                 -0.071      0.032     -2.198      0.028
204
205  Intercepts
206     MSQTOT1            0.000      0.000    999.000    999.000
207     MSQTOT4            0.000      0.000    999.000    999.000
208     MSQTOT7            0.000      0.000    999.000    999.000
209     I                  4.927      0.011    437.023      0.000
210     S                 -0.331      0.014    -24.437      0.000
211
```

```
 Residual Variances
    MSQTOT1            0.613      0.029     20.869      0.000
    MSQTOT4            0.808      0.017     48.451      0.000
    MSQTOT7            0.688      0.035     19.695      0.000
    I                  0.847      0.030     28.129      0.000
    S                  0.399      0.046      8.619      0.000


QUALITY OF NUMERICAL RESULTS

     Condition Number for the Information Matrix              0.379E-03
       (ratio of smallest to largest eigenvalue)


STANDARDIZED MODEL RESULTS


STDYX Standardization

                                                    Two-Tailed
                    Estimate       S.E.  Est./S.E.    P-Value

 I        |
    MSQTOT1            0.789      0.011     70.357      0.000
    MSQTOT4            0.714      0.009     76.654      0.000
    MSQTOT7            0.657      0.012     55.217      0.000

 S        |
    MSQTOT1            0.000      0.000    999.000    999.000
    MSQTOT4            0.284      0.014     19.852      0.000
    MSQTOT7            0.523      0.028     18.626      0.000

```

```
I        ON
   AGE                -0.294       0.010     -28.406       0.000
   BLACK              -0.280       0.010     -27.564       0.000

S        ON
   AGE                -0.308       0.024     -12.626       0.000
   BLACK              -0.065       0.018      -3.633       0.000

S        WITH
   I                  -0.121       0.048      -2.544       0.011

Intercepts
   MSQTOT1             0.000       0.000     999.000     999.000
   MSQTOT4             0.000       0.000     999.000     999.000
   MSQTOT7             0.000       0.000     999.000     999.000
   I                   4.910       0.077      63.670       0.000
   S                  -0.498       0.033     -14.921       0.000

Residual Variances
   MSQTOT1             0.378       0.018      21.402       0.000
   MSQTOT4             0.409       0.007      60.200       0.000
   MSQTOT7             0.295       0.014      20.376       0.000
   I                   0.841       0.008     101.310       0.000
   S                   0.902       0.015      59.102       0.000


STDY Standardization

                                                        Two-Tailed
                    Estimate       S.E.  Est./S.E.    P-Value

I        |
```

```
276     MSQTOT1            0.789      0.011     70.357      0.000
277     MSQTOT4            0.714      0.009     76.654      0.000
278     MSQTOT7            0.657      0.012     55.217      0.000
279
280  S        |
281     MSQTOT1            0.000      0.000    999.000    999.000
282     MSQTOT4            0.284      0.014     19.852      0.000
283     MSQTOT7            0.523      0.028     18.626      0.000
284
285  I        ON
286     AGE               -0.046      0.002    -28.754      0.000
287     BLACK             -0.705      0.025    -27.888      0.000
288
289  S        ON
290     AGE               -0.049      0.004    -12.655      0.000
291     BLACK             -0.163      0.045     -3.634      0.000
292
293  S        WITH
294     I                 -0.121      0.048     -2.544      0.011
295
296  Intercepts
297     MSQTOT1            0.000      0.000    999.000    999.000
298     MSQTOT4            0.000      0.000    999.000    999.000
299     MSQTOT7            0.000      0.000    999.000    999.000
300     I                  4.910      0.077     63.670      0.000
301     S                 -0.498      0.033    -14.921      0.000
302
303  Residual Variances
304     MSQTOT1            0.378      0.018     21.402      0.000
305     MSQTOT4            0.409      0.007     60.200      0.000
306     MSQTOT7            0.295      0.014     20.376      0.000
307     I                  0.841      0.008    101.310      0.000
```

```
    S                  0.902      0.015     59.102      0.000

STD Standardization

                                                    Two-Tailed
                     Estimate       S.E.  Est./S.E.    P-Value

 I        |
    MSQTOT1             1.003      0.016     64.168      0.000
    MSQTOT4             1.003      0.016     64.168      0.000
    MSQTOT7             1.003      0.016     64.168      0.000

 S        |
    MSQTOT1             0.000      0.000    999.000    999.000
    MSQTOT4             0.399      0.021     19.042      0.000
    MSQTOT7             0.798      0.042     19.042      0.000

 I        ON
    AGE                -0.046      0.002    -28.754      0.000
    BLACK              -0.705      0.025    -27.888      0.000

 S        ON
    AGE                -0.049      0.004    -12.655      0.000
    BLACK              -0.163      0.045     -3.634      0.000

 S        WITH
    I                  -0.121      0.048     -2.544      0.011

 Intercepts
    MSQTOT1             0.000      0.000    999.000    999.000
    MSQTOT4             0.000      0.000    999.000    999.000
```

```
340     MSQTOT7            0.000      0.000    999.000    999.000
341     I                  4.910      0.077     63.670      0.000
342     S                 -0.498      0.033    -14.921      0.000
343
344  Residual Variances
345     MSQTOT1            0.613      0.029     20.869      0.000
346     MSQTOT4            0.808      0.017     48.451      0.000
347     MSQTOT7            0.688      0.035     19.695      0.000
348     I                  0.841      0.008    101.310      0.000
349     S                  0.902      0.015     59.102      0.000
350
351
352 R-SQUARE
353
354     Observed                                        Two-Tailed
355     Variable        Estimate       S.E.  Est./S.E.    P-Value
356
357     MSQTOT1            0.622      0.018     35.179      0.000
358     MSQTOT4            0.591      0.007     86.894      0.000
359     MSQTOT7            0.705      0.014     48.680      0.000
360
361      Latent                                         Two-Tailed
362     Variable        Estimate       S.E.  Est./S.E.    P-Value
363
364     I                  0.159      0.008     19.156      0.000
365     S                  0.098      0.015      6.409      0.000
366
367
368 TECHNICAL 1 OUTPUT
369
370
371      PARAMETER SPECIFICATION
```

```
372
373
374             NU
375                MSQTOT1       MSQTOT4       MSQTOT7       AGE           BLACK
376                --------      --------      --------      --------      --------
377                   0             0             0             0             0
378
379
380             LAMBDA
381                I             S             AGE           BLACK
382                --------      --------      --------      --------
383   MSQTOT1           0             0             0             0
384   MSQTOT4           0             0             0             0
385   MSQTOT7           0             0             0             0
386   AGE               0             0             0             0
387   BLACK             0             0             0             0
388
389
390             THETA
391                MSQTOT1       MSQTOT4       MSQTOT7       AGE           BLACK
392                --------      --------      --------      --------      --------
393   MSQTOT1           1
394   MSQTOT4           0             2
395   MSQTOT7           0             0             3
396   AGE               0             0             0             0
397   BLACK             0             0             0             0             0
398
399
400             ALPHA
401                I             S             AGE           BLACK
402                --------      --------      --------      --------
403                   4             5             0             0
```

```
404
405
406             BETA
407                I             S             AGE           BLACK
408                --------      --------      --------      --------
409  I                   0             0             6             7
410  S                   0             0             8             9
411  AGE                 0             0             0             0
412  BLACK               0             0             0             0
413
414
415             PSI
416                I             S             AGE           BLACK
417                --------      --------      --------      --------
418  I                  10
419  S                  11            12
420  AGE                 0             0             0
421  BLACK               0             0             0             0
422
423
424       STARTING VALUES
425
426
427             NU
428                MSQTOT1       MSQTOT4       MSQTOT7       AGE           BLACK
429                --------      --------      --------      --------      --------
430                  0.000         0.000         0.000         0.000         0.000
431
432
433             LAMBDA
434                I             S             AGE           BLACK
435                --------      --------      --------      --------
```

```
436 MSQTOT1          1.000          0.000          0.000          0.000
437 MSQTOT4          1.000          0.600          0.000          0.000
438 MSQTOT7          1.000          1.200          0.000          0.000
439 AGE              0.000          0.000          1.000          0.000
440 BLACK            0.000          0.000          0.000          1.000
441
442
443             THETA
444                MSQTOT1        MSQTOT4        MSQTOT7        AGE            BLACK
445                --------       --------       --------       --------       --------
446 MSQTOT1          0.810
447 MSQTOT4          0.000          0.911
448 MSQTOT7          0.000          0.000          1.016
449 AGE              0.000          0.000          0.000          0.000
450 BLACK            0.000          0.000          0.000          0.000          0.000
451
452
453             ALPHA
454                I              S              AGE            BLACK
455                --------       --------       --------       --------
456                  4.948         -0.406          0.000          0.196
457
458
459             BETA
460                I              S              AGE            BLACK
461                --------       --------       --------       --------
462 I                0.000          0.000          0.000          0.000
463 S                0.000          0.000          0.000          0.000
464 AGE              0.000          0.000          0.000          0.000
465 BLACK            0.000          0.000          0.000          0.000
466
467
```

```
           PSI
              I             S             AGE           BLACK
              --------      --------      --------      --------
 I              1.247
 S              0.000         2.157
 AGE            0.000         0.000        40.275
 BLACK          0.000         0.000        -0.087         0.158


     Beginning Time:  13:40:45
        Ending Time:  13:40:45
       Elapsed Time:  00:00:00


Mplus VERSION 8.8 DEMO (Mac) has the following limitations:
  Maximum number of dependent variables: 6
  Maximum number of independent variables: 2
  Maximum number of between variables: 2
  Maximum number of continuous latent variables in time series analysis: 2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```

Step 3: Possible Model Modifications
Alternative Time Basis:
Age at Interview

Age at Interview

- Mplus allows the user to specify a LGM with individually varying time points
- Use TSCORES option
- Requires *time* as *data*
- Rather than *time* as a *parameter*

EPESE and Age

- Public use data provides age in groups
- For the sake of this example, randomly assign persons within an age group to an exact year within that group
- Center new pseudo-exact age variable at mean baseline age (grouped), $= 74.1$
- Divide by five (so s provides change per 5-year increment)
- LATER: Add quadratic function to capture non-linearity

An overly complicated approach to time/age

Actual Age Group at Baseline	Centered and 5-year Scaled Age at... Baseline	Wave 4	Wave 7
67: (65,69)	-1.82,-1.02	-1.22,-0.42	-0.62,0.18
72: (70,74)	-0.82,-0.02	-0.22, 0.58	0.38,1.18
77: (75,79)	0.18, 0.98	0.78, 1.58	1.38,2.18
82: (80,84)	1.18, 1.98	1.78, 2.58	2.38,3.18
87: (85,89)	2.18, 2.98	2.78, 3.58	3.38,4.18

ex0203.inp

```
TITLE:    TSCORES Approach to conditional LGM MSQ sum scores from EPESE
DATA:     FILE = ex0203.dat;
VARIABLE: NAMES = msqtot1 msqtot4 msqtot7 age1    age4    age7
                  male black ;
          MISSING  = ALL (-9999) ;

          TSCORES = age1 age4 age7 ;

DEFINE:   CENTER male black (GRANDMEAN) ;

OUTPUT:   TECH1 ;

ANALYSIS: TYPE = RANDOM ;

MODEL:    i s q | msqtot1 msqtot4 msqtot7 AT age1 age4 age7 ;
          q@0 ;
          i with q@0 ;
          s with q @0 ;
          i s q on male black ;
```

```
Mplus VERSION 8.8 DEMO (Mac)
MUTHEN & MUTHEN
07/11/2022   1:40 PM

INPUT INSTRUCTIONS

  TITLE:    TSCORES Approach to conditional LGM MSQ sum scores from EPESE
  DATA:     FILE = ex0203.dat;
  VARIABLE: NAMES = msqtot1 msqtot4 msqtot7 age1    age4    age7
                    male black ;
            MISSING  = ALL (-9999) ;

            TSCORES = age1 age4 age7 ;

  DEFINE:   CENTER male black (GRANDMEAN) ;

  OUTPUT:   TECH1 ;

  ANALYSIS: TYPE = RANDOM ;

  MODEL:    i s q | msqtot1 msqtot4 msqtot7 AT age1 age4 age7 ;
            q@0 ;
            i with q@0 ;
            s with q @0 ;
            i s q on male black ;
```

```
33
34 INPUT READING TERMINATED NORMALLY
35
36
37
38 TSCORES Approach to conditional LGM MSQ sum scores from EPESE
39
40 SUMMARY OF ANALYSIS
41
42 Number of groups                                                 1
43 Number of observations                                       14059
44
45 Number of dependent variables                                  3
46 Number of independent variables                                2
47 Number of continuous latent variables                          3
48
49 Observed dependent variables
50
51   Continuous
52    MSQTOT1     MSQTOT4     MSQTOT7
53
54 Observed independent variables
55    MALE        BLACK
56
57 Continuous latent variables
58    I           S           Q
59
60 Variables with special functions
61
62   Time scores
63    AGE1        AGE4        AGE7
64
```

```
  Centering (GRANDMEAN)
   MALE         BLACK


Estimator                                                MLR
Information matrix                                  OBSERVED
Maximum number of iterations                             100
Convergence criterion                               0.100D-05
Maximum number of EM iterations                          500
Convergence criteria for the EM algorithm
  Loglikelihood change                              0.100D-02
  Relative loglikelihood change                     0.100D-05
  Derivative                                        0.100D-03
Minimum variance                                    0.100D-03
Maximum number of steepest descent iterations            20
Maximum number of iterations for H1                    2000
Convergence criterion for H1                        0.100D-03
Optimization algorithm                                   EMA

Input data file(s)
  ex0203.dat
Input data format  FREE


SUMMARY OF DATA

     Number of missing data patterns             7


COVARIANCE COVERAGE OF DATA

Minimum covariance coverage value   0.100
```

```
 97
 98
 99      PROPORTION OF DATA PRESENT
100
101
102             Covariance Coverage
103                MSQTOT1       MSQTOT4       MSQTOT7       MALE          BLACK
104                --------      --------      --------      --------      --------
105  MSQTOT1         0.974
106  MSQTOT4         0.752         0.775
107  MSQTOT7         0.565         0.561         0.581
108  MALE            0.974         0.775         0.581         1.000
109  BLACK           0.974         0.775         0.581         1.000         1.000
110
111
112
113 UNIVARIATE SAMPLE STATISTICS
114
115
116      UNIVARIATE HIGHER-ORDER MOMENT DESCRIPTIVE STATISTICS
117
118          Variable/          Mean/     Skewness/   Minimum/ % with
                  Percentiles
119         Sample Size       Variance    Kurtosis    Maximum  Min/Max      20%/60%    40%/
                80%    Median
120
121      MSQTOT1                4.768       -1.152       0.000    0.70%       4.000
             5.000       5.000
122            13698.000        1.621        1.156       6.000   34.84%       5.000
                   6.000
123      MSQTOT4                4.732       -1.233       0.000    1.17%       4.000
             5.000       5.000
```

```
124              10899.000        1.822        1.268        6.000    35.05%        5.000
                       6.000
125        MSQTOT7                4.569       -1.160        0.000     2.00%        4.000
             5.000       5.000
126              8169.000        2.033        1.060        6.000    30.31%        5.000
                       6.000
127        MALE                   0.000        0.505       -0.378    62.24%       -0.378
             -0.378      -0.378
128              14059.000        0.235       -1.745        0.622    37.76%       -0.378
                       0.622
129        BLACK                  0.000        1.528       -0.196    80.35%       -0.196
             -0.196      -0.196
130              14059.000        0.158        0.335        0.804    19.65%       -0.196
                       -0.196
131
132
133 THE MODEL ESTIMATION TERMINATED NORMALLY
134
135
136
137 MODEL FIT INFORMATION
138
139 Number of Free Parameters                        15
140
141 Loglikelihood
142
143           H0 Value                        -51096.031
144           H0 Scaling Correction Factor       1.2884
145             for MLR
146
147 Information Criteria
148
```

```
          Akaike (AIC)                    102222.062
          Bayesian (BIC)                  102335.327
          Sample-Size Adjusted BIC        102287.659
            (n* = (n + 2) / 24)



MODEL RESULTS

                                                    Two-Tailed
                    Estimate       S.E.  Est./S.E.    P-Value

 I          ON
    MALE              0.084      0.021      3.917      0.000
    BLACK            -0.687      0.031    -22.471      0.000

 S          ON
    MALE              0.023      0.015      1.524      0.127
    BLACK            -0.049      0.020     -2.421      0.015

 Q          ON
    MALE             -0.001      0.009     -0.123      0.902
    BLACK            -0.027      0.011     -2.407      0.016

 I        WITH
    Q                 0.000      0.000    999.000    999.000

 S        WITH
    Q                 0.000      0.000    999.000    999.000
    I                 0.114      0.008     13.932      0.000

 Intercepts
```

```
181     MSQTOT1              0.000       0.000     999.000     999.000
182     MSQTOT4              0.000       0.000     999.000     999.000
183     MSQTOT7              0.000       0.000     999.000     999.000
184     I                    4.885       0.011     464.207       0.000
185     S                   -0.205       0.007     -27.955       0.000
186     Q                   -0.085       0.004     -19.380       0.000
187
188  Residual Variances
189     MSQTOT1              0.729       0.017      43.958       0.000
190     MSQTOT4              0.776       0.019      41.802       0.000
191     MSQTOT7              0.917       0.027      33.986       0.000
192     I                    0.622       0.021      29.709       0.000
193     S                    0.089       0.009       9.404       0.000
194     Q                    0.000       0.000     999.000     999.000
195
196
197 QUALITY OF NUMERICAL RESULTS
198
199      Condition Number for the Information Matrix              0.251E-03
200        (ratio of smallest to largest eigenvalue)
201
202
203 TECHNICAL 1 OUTPUT
204
205
206      PARAMETER SPECIFICATION
207
208
209             NU
210                MSQTOT1       MSQTOT4       MSQTOT7       MALE          BLACK
211                --------      --------      --------      --------      --------
212                       0             0             0             0             0
```

```
213
214
215             LAMBDA
216                I             S             Q             MALE          BLACK
217                ________      ________      ________      ________      ________
218  MSQTOT1             0             0             0             0             0
219  MSQTOT4             0             0             0             0             0
220  MSQTOT7             0             0             0             0             0
221  MALE                0             0             0             0             0
222  BLACK               0             0             0             0             0
223
224
225             THETA
226                MSQTOT1       MSQTOT4       MSQTOT7       MALE          BLACK
227                ________      ________      ________      ________      ________
228  MSQTOT1             1
229  MSQTOT4             0             2
230  MSQTOT7             0             0             3
231  MALE                0             0             0             0
232  BLACK               0             0             0             0             0
233
234
235             ALPHA
236                I             S             Q             MALE          BLACK
237                ________      ________      ________      ________      ________
238                       4             5             6             0             0
239
240
241             BETA
242                I             S             Q             MALE          BLACK
243                ________      ________      ________      ________      ________
244  I                   0             0             0             7             8
```

```
S                  0              0              0              9             10
Q                  0              0              0             11             12
MALE               0              0              0              0              0
BLACK              0              0              0              0              0


           PSI
              I              S              Q              MALE           BLACK
              ________       ________       ________       ________       ________
I                  13
S                  14             15
Q                   0              0              0
MALE                0              0              0              0
BLACK               0              0              0              0              0


     STARTING VALUES


           NU
              MSQTOT1        MSQTOT4        MSQTOT7        MALE           BLACK
              ________       ________       ________       ________       ________
                0.000          0.000          0.000          0.000          0.000


           LAMBDA
              I              S              Q              MALE           BLACK
              ________       ________       ________       ________       ________
MSQTOT1         0.000          0.000          0.000          0.000          0.000
MSQTOT4         0.000          0.000          0.000          0.000          0.000
MSQTOT7         0.000          0.000          0.000          0.000          0.000
MALE            0.000          0.000          0.000          1.000          0.000
```

```
277  BLACK              0.000          0.000          0.000          0.000          1.000
278
279
280             THETA
281                MSQTOT1        MSQTOT4        MSQTOT7        MALE           BLACK
282                --------       --------       --------       --------       --------
283  MSQTOT1         0.810
284  MSQTOT4         0.000          0.911
285  MSQTOT7         0.000          0.000          1.016
286  MALE            0.000          0.000          0.000          0.000
287  BLACK           0.000          0.000          0.000          0.000          0.000
288
289
290             ALPHA
291                I              S              Q              MALE           BLACK
292                --------       --------       --------       --------       --------
293                4.958          -0.273         -0.318         0.000          0.000
294
295
296             BETA
297                I              S              Q              MALE           BLACK
298                --------       --------       --------       --------       --------
299  I             0.000          0.000          0.000          0.000          0.000
300  S             0.000          0.000          0.000          0.000          0.000
301  Q             0.000          0.000          0.000          0.000          0.000
302  MALE          0.000          0.000          0.000          0.000          0.000
303  BLACK         0.000          0.000          0.000          0.000          0.000
304
305
306             PSI
307                I              S              Q              MALE           BLACK
308                --------       --------       --------       --------       --------
```

```
 I                1.504
 S                0.000          5.568
 Q                0.000          0.000          0.000
 MALE             0.000          0.000          0.000          0.118
 BLACK            0.000          0.000          0.000          0.000          0.079


     Beginning Time:  13:40:46
        Ending Time:  13:40:59
       Elapsed Time:  00:00:13


Mplus VERSION 8.8 DEMO (Mac) has the following limitations:
  Maximum number of dependent variables: 6
  Maximum number of independent variables: 2
  Maximum number of between variables: 2
  Maximum number of continuous latent variables in time series analysis: 2


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```

ex0203.out

Add a Quadratic Effect

- Previously, acceleration of change associated with advanced age was modeled with a linear regression of s on age (linear LGM)
- We could capture non-linear acceleration with an age^2 term or dummy indicator variables for age groups
- In the random effects framework, we can include a time factor with quadratic time steps

ex0204.inp

```
TITLE:    TSCORES Approach to conditional LGM MSQ sum scores from EPESE
          Random effects, age basis, with quadratic age effect ;

DATA:     FILE = ex0203.dat;

VARIABLE: NAMES = msqtot1 msqtot4 msqtot7 age1    age4    age7
                  male black ;
          usevariables = msqtot1 msqtot4 msqtot7 male black ;
          MISSING  = ALL (-9999) ;
          !TSCORES = age1 age4 age7 ;

DEFINE:   CENTER male black (GRANDMEAN) ;

OUTPUT:   TECH1 ;

!ANALYSIS: TYPE = RANDOM ;

MODEL:    !i s q | msqtot1 msqtot4 msqtot7 AT age1 age4 age7 ;
          i s q | msqtot1@0 msqtot4@0.6 msqtot7@1.2; ! AT age1 age4 age7 ;
          i s q on male black ;
          msqtot1-msqtot7 (1) ; ! required for identification
```

ex0204.inp , continued

```
            q@0; ! quadratic effect is a fixed effect


    SAVEDATA: SAVE = fscores ; file=ex0203fs.dat ;
```

```
Mplus VERSION 8.8 DEMO (Mac)
MUTHEN & MUTHEN
07/11/2022   1:40 PM

INPUT INSTRUCTIONS

  TITLE:    TSCORES Approach to conditional LGM MSQ sum scores from EPESE
            Random effects, age basis, with quadratic age effect ;

  DATA:     FILE = ex0203.dat;

  VARIABLE: NAMES = msqtot1 msqtot4 msqtot7 age1    age4    age7
                    male black ;
            usevariables = msqtot1 msqtot4 msqtot7 male black ;
            MISSING  = ALL (-9999) ;
            !TSCORES = age1 age4 age7 ;

  DEFINE:   CENTER male black (GRANDMEAN) ;

  OUTPUT:   TECH1 ;

  !ANALYSIS: TYPE = RANDOM ;

  MODEL:    !i s q | msqtot1 msqtot4 msqtot7 AT age1 age4 age7 ;
            i s q | msqtot1@0 msqtot4@0.6 msqtot7@1.2; ! AT age1 age4 age7 ;
            i s q on male black ;
            msqtot1-msqtot7 (1) ; ! required for identification
            q@0; ! quadratic effect is a fixed effect


   SAVEDATA: SAVE = fscores ; file=ex0203fs.dat ;

```

```
33
34
35 *** WARNING in MODEL command
36   All continuous latent variable covariances involving Q have been fixed to 0
37   because the variance of Q is fixed at 0.
38    1 WARNING(S) FOUND IN THE INPUT INSTRUCTIONS
39
40
41
42 TSCORES Approach to conditional LGM MSQ sum scores from EPESE
43 Random effects, age basis, with quadratic age effect ;
44
45 SUMMARY OF ANALYSIS
46
47 Number of groups                                                 1
48 Number of observations                                     14059
49
50 Number of dependent variables                                  3
51 Number of independent variables                                2
52 Number of continuous latent variables                          3
53
54 Observed dependent variables
55
56   Continuous
57    MSQTOT1     MSQTOT4     MSQTOT7
58
59 Observed independent variables
60    MALE        BLACK
61
62 Continuous latent variables
63    I           S           Q
64
```

```
Variables with special functions

  Centering (GRANDMEAN)
   MALE         BLACK


Estimator                                                      ML
Information matrix                                       OBSERVED
Maximum number of iterations                                 1000
Convergence criterion                                  0.500D-04
Maximum number of steepest descent iterations                  20
Maximum number of iterations for H1                          2000
Convergence criterion for H1                           0.100D-03

Input data file(s)
  ex0203.dat

Input data format  FREE


SUMMARY OF DATA

     Number of missing data patterns             7


COVARIANCE COVERAGE OF DATA

Minimum covariance coverage value   0.100


     PROPORTION OF DATA PRESENT

```

```
 97
 98              Covariance Coverage
 99                 MSQTOT1        MSQTOT4        MSQTOT7        MALE           BLACK
100                 --------       --------       --------       --------       --------
101  MSQTOT1          0.974
102  MSQTOT4          0.752          0.775
103  MSQTOT7          0.565          0.561          0.581
104  MALE             0.974          0.775          0.581          1.000
105  BLACK            0.974          0.775          0.581          1.000          1.000
106
107
108
109 UNIVARIATE SAMPLE STATISTICS
110
111
112      UNIVARIATE HIGHER-ORDER MOMENT DESCRIPTIVE STATISTICS
113
114          Variable/          Mean/     Skewness/   Minimum/ % with
                  Percentiles
115         Sample Size      Variance    Kurtosis    Maximum  Min/Max      20%/60%    40%/
                80%    Median
116
117      MSQTOT1                4.768       -1.152       0.000    0.70%       4.000
             5.000       5.000
118           13698.000         1.621        1.156       6.000   34.84%       5.000
                  6.000
119      MSQTOT4                4.732       -1.233       0.000    1.17%       4.000
             5.000       5.000
120           10899.000         1.822        1.268       6.000   35.05%       5.000
                  6.000
121      MSQTOT7                4.569       -1.160       0.000    2.00%       4.000
             5.000       5.000
```

```
122              8169.000        2.033        1.060        6.000    30.31%        5.000
                      6.000
123       MALE                   0.000        0.505       -0.378    62.24%       -0.378
             -0.378      -0.378
124              14059.000        0.235       -1.745        0.622    37.76%       -0.378
                      0.622
125       BLACK                  0.000        1.528       -0.196    80.35%       -0.196
             -0.196      -0.196
126              14059.000        0.158        0.335        0.804    19.65%       -0.196
                     -0.196
127
128
129 THE MODEL ESTIMATION TERMINATED NORMALLY
130
131
132
133 MODEL FIT INFORMATION
134
135 Number of Free Parameters                        13
136
137 Loglikelihood
138
139           H0 Value                         -52205.932
140           H1 Value                         -52195.142
141
142 Information Criteria
143
144           Akaike (AIC)                     104437.864
145           Bayesian (BIC)                   104536.027
146           Sample-Size Adjusted BIC         104494.714
147             (n* = (n + 2) / 24)
148
```

```
Chi-Square Test of Model Fit

          Value                             21.579
          Degrees of Freedom                    2
          P-Value                           0.0000

RMSEA (Root Mean Square Error Of Approximation)

          Estimate                           0.026
          90 Percent C.I.                    0.017  0.037
          Probability RMSEA <= .05           1.000

CFI/TLI

          CFI                                0.997
          TLI                                0.988

Chi-Square Test of Model Fit for the Baseline Model

          Value                           7541.543
          Degrees of Freedom                    9
          P-Value                           0.0000

SRMR (Standardized Root Mean Square Residual)

          Value                              0.024


MODEL RESULTS

                                                    Two-Tailed
```

```
                    Estimate       S.E.  Est./S.E.    P-Value

I        |
    MSQTOT1            1.000      0.000    999.000    999.000
    MSQTOT4            1.000      0.000    999.000    999.000
    MSQTOT7            1.000      0.000    999.000    999.000

S        |
    MSQTOT1            0.000      0.000    999.000    999.000
    MSQTOT4            0.600      0.000    999.000    999.000
    MSQTOT7            1.200      0.000    999.000    999.000

Q        |
    MSQTOT1            0.000      0.000    999.000    999.000
    MSQTOT4            0.360      0.000    999.000    999.000
    MSQTOT7            1.440      0.000    999.000    999.000

I        ON
    MALE               0.110      0.022      4.977      0.000
    BLACK             -0.696      0.027    -26.076      0.000

S        ON
    MALE               0.047      0.075      0.626      0.531
    BLACK              0.201      0.090      2.239      0.025

Q        ON
    MALE               0.049      0.062      0.791      0.429
    BLACK             -0.274      0.074     -3.687      0.000

S        WITH
    I                  0.088      0.020      4.463      0.000

```

```
 Intercepts
    MSQTOT1            0.000      0.000    999.000    999.000
    MSQTOT4            0.000      0.000    999.000    999.000
    MSQTOT7            0.000      0.000    999.000    999.000
    I                  4.770      0.011    447.662      0.000
    S                 -0.031      0.036     -0.851      0.395
    Q                 -0.241      0.030     -8.085      0.000

 Residual Variances
    MSQTOT1            0.753      0.012     64.107      0.000
    MSQTOT4            0.753      0.012     64.107      0.000
    MSQTOT7            0.753      0.012     64.107      0.000
    I                  0.813      0.021     38.925      0.000
    S                  0.296      0.029     10.225      0.000
    Q                  0.000      0.000    999.000    999.000


QUALITY OF NUMERICAL RESULTS

     Condition Number for the Information Matrix              0.118E-02
       (ratio of smallest to largest eigenvalue)


TECHNICAL 1 OUTPUT


     PARAMETER SPECIFICATION


           NU
              MSQTOT1       MSQTOT4       MSQTOT7       MALE          BLACK
              --------      --------      --------      --------      --------
```

```
                    0             0             0             0             0

           LAMBDA
              I             S             Q             MALE          BLACK
              --------      --------      --------      --------      --------
MSQTOT1             0             0             0             0             0
MSQTOT4             0             0             0             0             0
MSQTOT7             0             0             0             0             0
MALE                0             0             0             0             0
BLACK               0             0             0             0             0


           THETA
              MSQTOT1       MSQTOT4       MSQTOT7       MALE          BLACK
              --------      --------      --------      --------      --------
MSQTOT1             1
MSQTOT4             0             1
MSQTOT7             0             0             1
MALE                0             0             0             0
BLACK               0             0             0             0             0


           ALPHA
              I             S             Q             MALE          BLACK
              --------      --------      --------      --------      --------
                  2             3             4             0             0


           BETA
              I             S             Q             MALE          BLACK
              --------      --------      --------      --------      --------
```

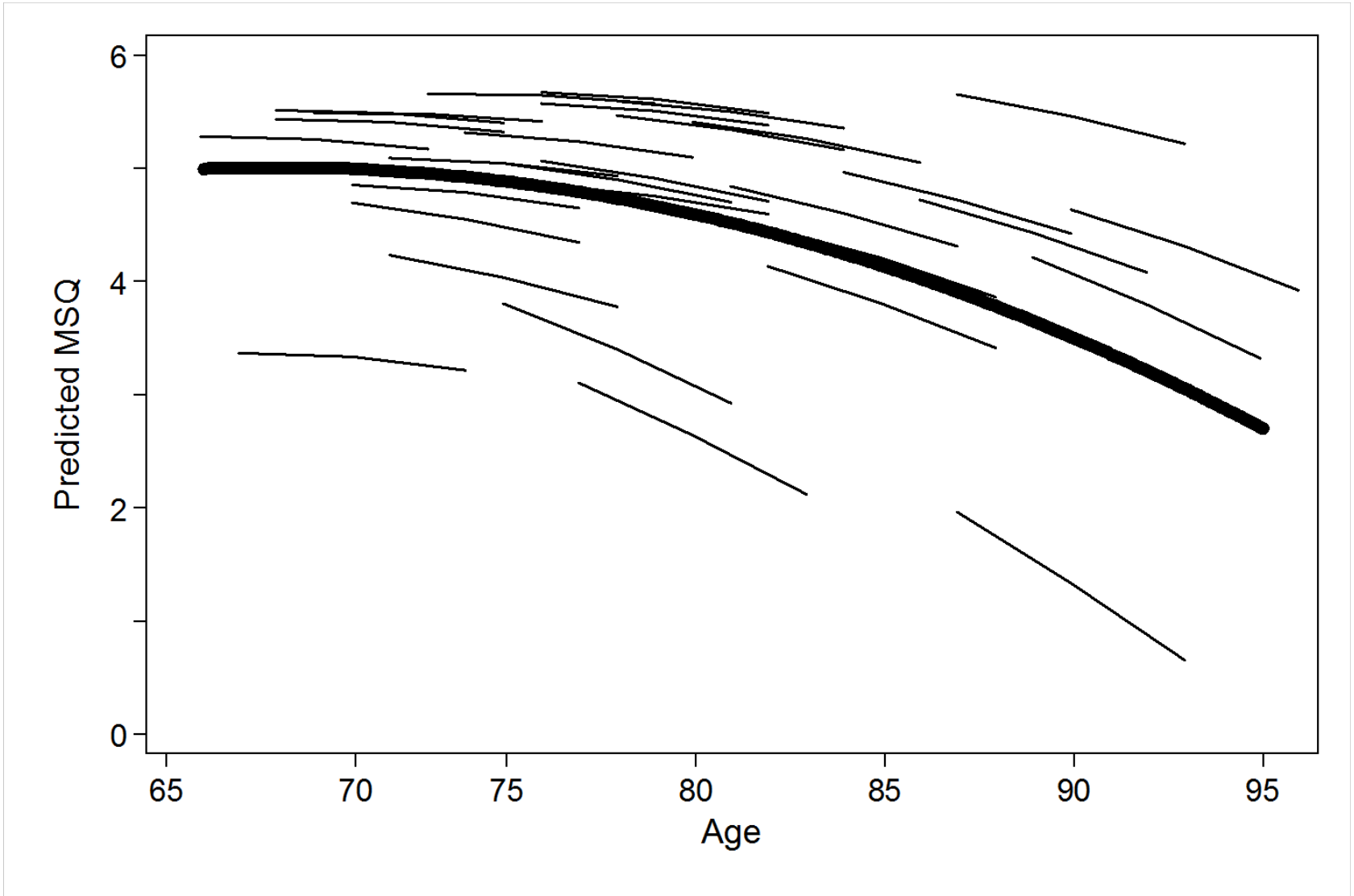
```
277 I                  0              0              0              5              6
278 S                  0              0              0              7              8
279 Q                  0              0              0              9             10
280 MALE               0              0              0              0              0
281 BLACK              0              0              0              0              0
282
283
284            PSI
285               I             S             Q             MALE          BLACK
286               --------      --------      --------      --------      --------
287 I                  11
288 S                  12            13
289 Q                   0             0             0
290 MALE                0             0             0             0
291 BLACK               0             0             0             0              0
292
293
294        STARTING VALUES
295
296
297            NU
298               MSQTOT1       MSQTOT4       MSQTOT7       MALE          BLACK
299               --------      --------      --------      --------      --------
300                 0.000         0.000         0.000         0.000         0.000
301
302
303            LAMBDA
304               I             S             Q             MALE          BLACK
305               --------      --------      --------      --------      --------
306 MSQTOT1         1.000         0.000         0.000         0.000         0.000
307 MSQTOT4         1.000         0.600         0.360         0.000         0.000
308 MSQTOT7         1.000         1.200         1.440         0.000         0.000
```

```
MALE              0.000         0.000         0.000         1.000         0.000
BLACK             0.000         0.000         0.000         0.000         1.000


           THETA
              MSQTOT1       MSQTOT4       MSQTOT7       MALE          BLACK
              --------      --------      --------      --------      --------
MSQTOT1         0.810
MSQTOT4         0.000         0.911
MSQTOT7         0.000         0.000         1.016
MALE            0.000         0.000         0.000         0.000
BLACK           0.000         0.000         0.000         0.000         0.000


           ALPHA
              I             S             Q             MALE          BLACK
              --------      --------      --------      --------      --------
                4.985         0.123        -0.381         0.000         0.000


           BETA
              I             S             Q             MALE          BLACK
              --------      --------      --------      --------      --------
I               0.000         0.000         0.000         0.000         0.000
S               0.000         0.000         0.000         0.000         0.000
Q               0.000         0.000         0.000         0.000         0.000
MALE            0.000         0.000         0.000         0.000         0.000
BLACK           0.000         0.000         0.000         0.000         0.000


           PSI
              I             S             Q             MALE          BLACK
```

```
                --------      --------      --------      --------      --------
 I                1.178
 S                0.000        11.337
 Q                0.000         0.000         0.000
 MALE             0.000         0.000         0.000         0.235
 BLACK            0.000         0.000         0.000        -0.006         0.158


SAMPLE STATISTICS FOR ESTIMATED FACTOR SCORES


     SAMPLE STATISTICS


           Means
              I             I_SE          S             S_SE          Q
              --------      --------      --------      --------      --------
                4.770         0.512        -0.031         0.476        -0.241


           Means
              Q_SE
              --------
                0.000


           Covariances
              I             I_SE          S             S_SE          Q
              --------      --------      --------      --------      --------
 I              0.627
 I_SE          -0.009         0.004
 S              0.108        -0.002         0.075
```

```
 S_SE           -0.007          0.002         -0.001          0.002
 Q               0.032          0.000         -0.008          0.000          0.013
 Q_SE            0.000          0.000          0.000          0.000          0.000


           Covariances
              Q_SE
              --------
 Q_SE           0.000


           Correlations
              I              I_SE           S              S_SE           Q
              --------       --------       --------       --------       --------
 I              1.000
 I_SE          -0.182          1.000
 S              0.501         -0.088          1.000
 S_SE          -0.216          0.895         -0.125          1.000
 Q              0.357          0.037         -0.266          0.031          1.000
 Q_SE         999.000        999.000        999.000        999.000        999.000


           Correlations
              Q_SE
              --------
 Q_SE           1.000


SAVEDATA INFORMATION


  Save file
```

```
405     ex0203fs.dat
406
407   Order and format of variables
408
409     MSQTOT1         F10.3
410     MSQTOT4         F10.3
411     MSQTOT7         F10.3
412     MALE            F10.3
413     BLACK           F10.3
414     I               F10.3
415     I_SE            F10.3
416     S               F10.3
417     S_SE            F10.3
418     Q               F10.3
419     Q_SE            F10.3
420
421   Save file format
422     11F10.3
423
424   Save file record length    10000
425
426   Save missing symbol        *
427
428
429      Beginning Time:  13:40:59
430         Ending Time:  13:41:00
431        Elapsed Time:  00:00:01
432
433
434 Mplus VERSION 8.8 DEMO (Mac) has the following limitations:
435   Maximum number of dependent variables: 6
436   Maximum number of independent variables: 2
```


Predicted MSQ
6
4
2
0
65
70
75
80
85
90
95
Age

Sharing Results

Describing the LGM

Table 1. Subject Characteristics.

Characteristic	Mean or n	(SD) or (%)	Observed range
Total [n (%)]	14,059	(100)	
Age [M (SD)]	73.9	(6.5)	[65.0–89.0]
Sex [n (%)]			
Women	8,751	(62.2)	
Men	5,308	(37.8)	
Black or African-American [n (%)]			
No	11,297	(80.4)	
Yes	2,762	(19.6)	
MSQ			
visit 1 [M (SD)]	4.8	(1.3)	[0.0–6.0]
visit 4 [M (SD)]	4.7	(1.3)	[0.0–6.0]
visit 7 [M (SD)]	4.6	(1.4)	[0.0–6.0]

Table 2. Model Results.

Parameter	Model 1 Estimate	(SE)	Model 2 Estimate	(SE)
Level (at age 74)	4.77	0.01	4.78	0.01
Slope (per 5 years)	-0.26	0.01	-0.26	0.01
Variance(Level)	0.71	0.02	0.62	0.02
Variance(Slope)	0.09	0.01	0.08	0.01
Covariance(Level,Slope)	0.11	0.01	0.11	0.01
Level on				
Black or AA	-	-	-0.73	0.03
Male	-	-	0.08	0.02
Slope on				
Black or AA	-	-	-0.06	0.02
Male	-	-	0.05	0.01
Residual Variances				
visit 1	0.75	0.02	0.75	0.01
visit 4	0.77	0.02	0.79	0.01
visit 7	0.96	0.03	0.95	0.01
Empirical r^2	0.74		0.73	

Empirical r^2

- Random effect models (MLR with TSCORES) do not produce CFI, RMSEA
- Singer and Willet (2003) propose empirical r-square [1] as a measure of fit
- Calculate predicted values for each person based on estimated model parameters
- Report the squared estimate correlation of predicted and observed

[1] But, they call it a *pseudo-r-squared*. I think *empirical* captures the same meaning but is less pejorative

Obtaining Empirical r^2

- Use SAVEDATA option to save factor scores (i and s)
- Import ASCII data into your favorite stats package
- Calculate predicted values from i and s (and not x)
- Estimate correlation with observed values
- Square it

ex0205.inp

```
TITLE:    TSCORES Approach to conditional LGM MSQ sum scores from EPESE
          Random effects, age basis, and SAVEDATA command

DATA:     FILE = ex0205.dat; ! note new data file

VARIABLE: NAMES = msqtot1 msqtot4 msqtot7 age1    age4    age7
                  male black
                  id ; ! idvariable added
          MISSING  = ALL (-9999) ;
          TSCORES = age1 age4 age7 ;
          IDVARIABLE = id ; ! identify the ID variable

DEFINE:   CENTER male black (GRANDMEAN) ;

OUTPUT:   TECH1 ;

SAVEDATA: SAVE = FSCORES ;       ! What to save
          FILE = fscores.dat  ;  ! Name the file

ANALYSIS: TYPE = RANDOM ;
```

ex0205.inp , continued

```
MODEL:    i s | msqtot1 msqtot4 msqtot7 AT age1 age4 age7 ;
          i s on male black ;
          msqtot1-msqtot7 ;
```

```
* STATA CODE
* unconditional model
save workingdata , replace
runmplus msq1 msq4 msq7 age1 age4 age7 id , ///
 idvariable(id) tscores(age1 age4 age7 ) ///
 analysis(type=random ;)
 model(i s | msq1 msq4 msq7 at age1 age4 age7;) ///
 savedata(save=fscores; file=model1.dat) ///
 savelogfile(model1)
runmplus_load_savedata , out(model1.out) clear
keep id i s
merge id using workingdata , sort
reshape long msqtot age , i(id) j(wave)
generate py = I + s * age
summarize msqtot py
correlate msqtot py
di "empirical r-square = " r(rho)^2
```

Lab Session
LGM

Growth Mixture Model

Evaluate different subpopulations with distinct change trajectories

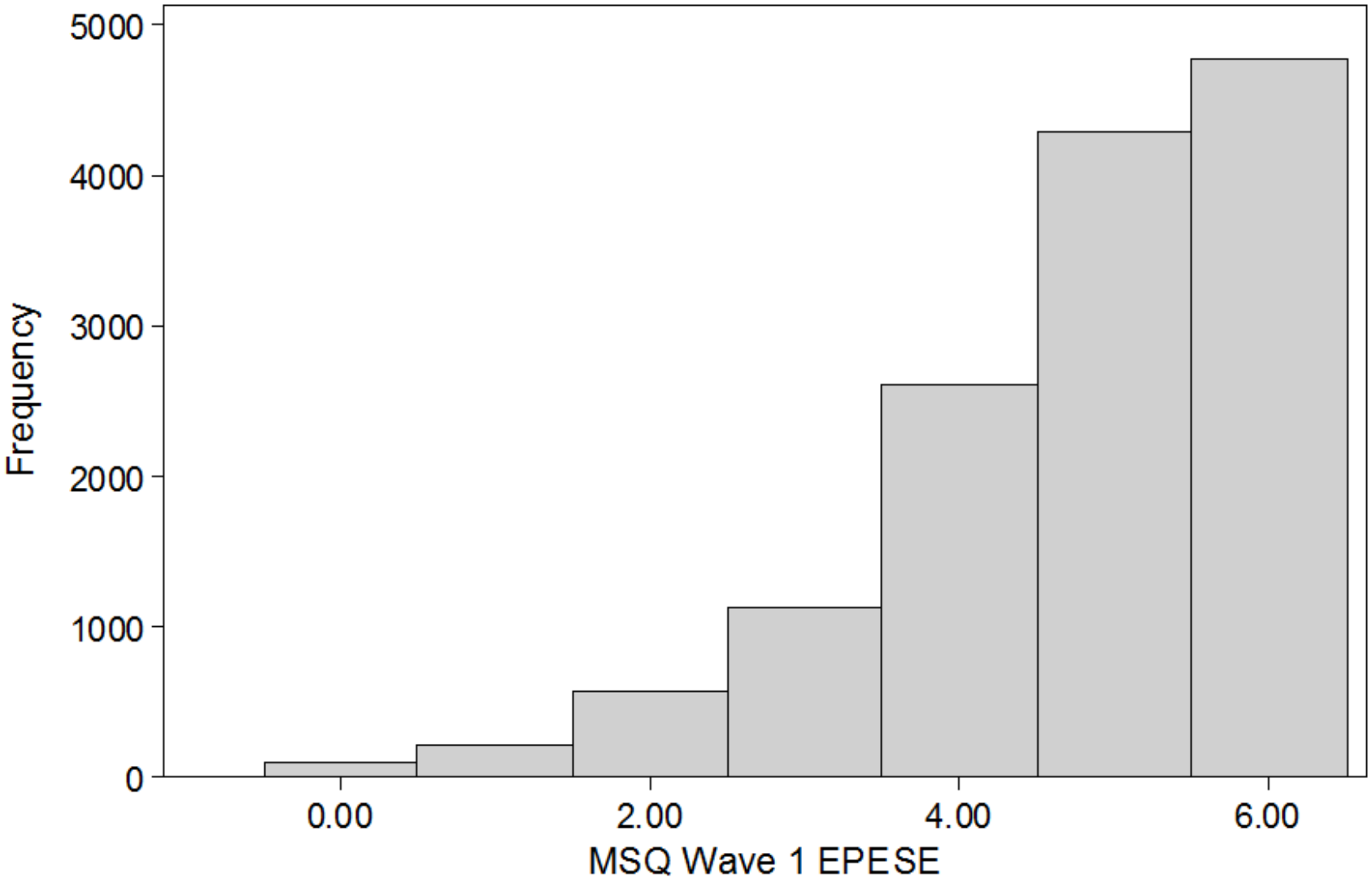
Growth Mixture Model

- Speculate that the population is comprised of distinct subpopulations
- Defined by different patterns of change
- Use latent class (profile mixture) model superimposed on latent growth curve model

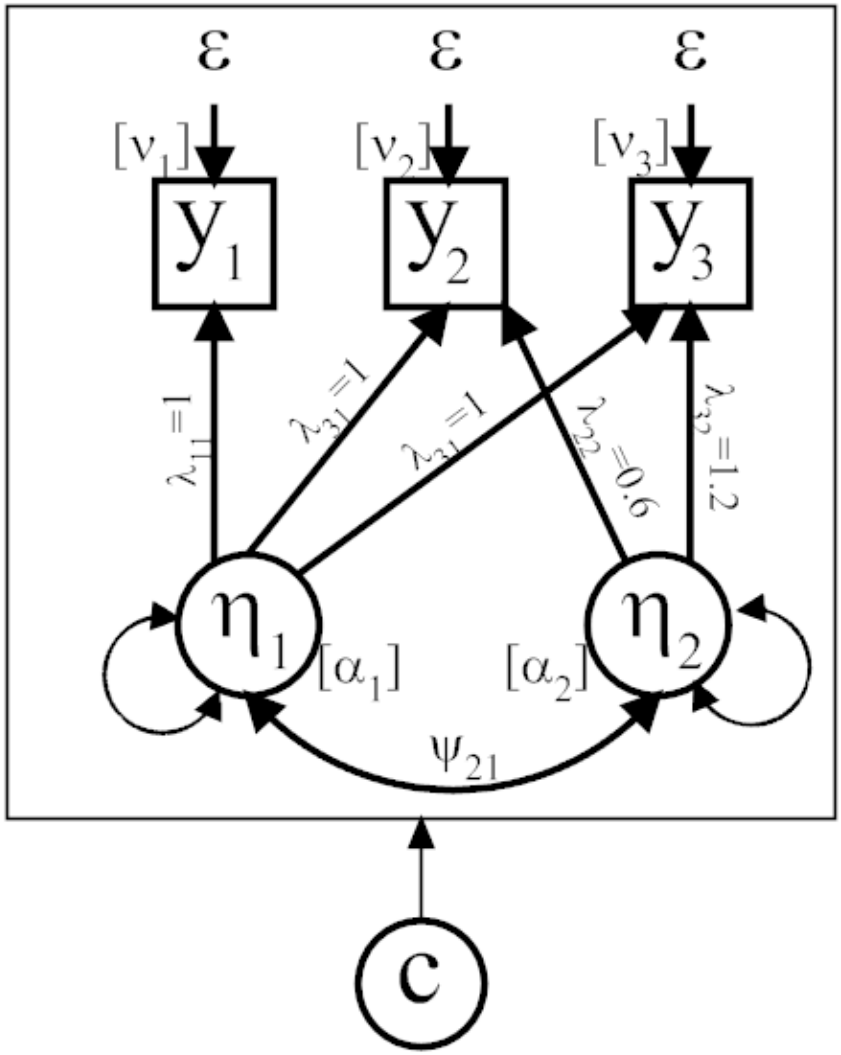
Other Resources

Jung, T. & Wickrama, K. A. S. (2008). An introduction to latent class growth analysis and growth mixture modeling. Social and Personality Psychology Compass, 2(1), 302-317

http://statmodel2.com/download/JungWickramaLCGALGMM.pdf


5000
4000
3000
2000
1000
0
Frequency
0.00
2.00
4.00
6.00
MSQ Wave 1 EPESE

Growth Mixture Model



ex0206.inp

```
TITLE:    Growth mixture model, MSQ sum scores
DATA:     FILE = ex0201.dat;
VARIABLE: NAMES = msqtot1 msqtot4 msqtot7 ;
          MISSING  = ALL (-9999) ;
          CLASSES = c(2) ; ! name class var & define # of levels
OUTPUT:   TECH1 ;
 TECH14;
          TECH11 ;
ANALYSIS: TYPE = MIXTURE ;  ! ask for a mixture model
MODEL:    %OVERALL%
          i s | msqtot1@0 msqtot4@.6 msqtot7@1.2 ;
          %c#1%
          !msqtot1-msqtot7 (1) ;
          [i ]; i ;
          [s ] ;
          !i s; i with s ;
          %c#2%
          !msqtot1-msqtot7 (2) ;
          [i ]; i ;
          [s ] ;
          ! i s; i with s ;
```

```
Mplus VERSION 8.8 DEMO (Mac)
MUTHEN & MUTHEN
07/11/2022   1:41 PM

INPUT INSTRUCTIONS

  TITLE:    Growth mixture model, MSQ sum scores
  DATA:     FILE = ex0201.dat;
  VARIABLE: NAMES = msqtot1 msqtot4 msqtot7 ;
            MISSING  = ALL (-9999) ;
            CLASSES = c(2) ; ! name class var & define # of levels
  OUTPUT:   TECH1 ;
         TECH14;
            TECH11 ;
  ANALYSIS: TYPE = MIXTURE ;  ! ask for a mixture model
  MODEL:    %OVERALL%
            i s | msqtot1@0 msqtot4@.6 msqtot7@1.2 ;
            %c#1%
            !msqtot1-msqtot7 (1) ;
            [i ]; i ;
            [s ] ;
            !i s; i with s ;
            %c#2%
            !msqtot1-msqtot7 (2) ;
            [i ]; i ;
            [s ] ;
            ! i s; i with s ;
```

```
*** WARNING
  Data set contains cases with missing on all variables.
  These cases were not included in the analysis.
  Number of cases with missing on all variables:  397
   1 WARNING(S) FOUND IN THE INPUT INSTRUCTIONS



Growth mixture model, MSQ sum scores

SUMMARY OF ANALYSIS

Number of groups                                                 1
Number of observations                                       14059

Number of dependent variables                                    3
Number of independent variables                                  0
Number of continuous latent variables                            2
Number of categorical latent variables                           1

Observed dependent variables

  Continuous
   MSQTOT1     MSQTOT4     MSQTOT7

Continuous latent variables
   I           S

Categorical latent variables
   C


```

```
Estimator                                                      MLR
Information matrix                                        OBSERVED
Optimization Specifications for the Quasi-Newton Algorithm for
Continuous Outcomes
  Maximum number of iterations                                 100
  Convergence criterion                                 0.100D-05
Optimization Specifications for the EM Algorithm
  Maximum number of iterations                                 500
  Convergence criteria
    Loglikelihood change                                0.100D-06
    Relative loglikelihood change                       0.100D-06
    Derivative                                          0.100D-05
Optimization Specifications for the M step of the EM Algorithm for
Categorical Latent variables
  Number of M step iterations                                    1
  M step convergence criterion                          0.100D-05
  Basis for M step termination                          ITERATION
Optimization Specifications for the M step of the EM Algorithm for
Censored, Binary or Ordered Categorical (Ordinal), Unordered
Categorical (Nominal) and Count Outcomes
  Number of M step iterations                                    1
  M step convergence criterion                          0.100D-05
  Basis for M step termination                          ITERATION
  Maximum value for logit thresholds                           15
  Minimum value for logit thresholds                          -15
  Minimum expected cell size for chi-square             0.100D-01
Maximum number of iterations for H1                           2000
Convergence criterion for H1                            0.100D-03
Optimization algorithm                                         EMA
Random Starts Specifications
  Number of initial stage random starts                        20
  Number of final stage optimizations                           4
```

```
  Number of initial stage iterations                    10
  Initial stage convergence criterion              0.100D+01
  Random starts scale                              0.500D+01
  Random seed for generating random starts               0

Input data file(s)
  ex0201.dat
Input data format  FREE


SUMMARY OF DATA

     Number of missing data patterns             7
     Number of y missing data patterns           7
     Number of u missing data patterns           0


COVARIANCE COVERAGE OF DATA

Minimum covariance coverage value   0.100


     PROPORTION OF DATA PRESENT FOR Y


           Covariance Coverage
              MSQTOT1       MSQTOT4       MSQTOT7
              --------      --------      --------
 MSQTOT1        0.974
 MSQTOT4        0.752         0.775
 MSQTOT7        0.565         0.561         0.581

```

```
129
130
131 UNIVARIATE SAMPLE STATISTICS
132
133
134      UNIVARIATE HIGHER-ORDER MOMENT DESCRIPTIVE STATISTICS
135
136          Variable/          Mean/     Skewness/   Minimum/ % with
                  Percentiles
137         Sample Size      Variance    Kurtosis    Maximum  Min/Max      20%/60%    40%/
                80%    Median
138
139      MSQTOT1                4.768       -1.152       0.000    0.70%       4.000
            5.000       5.000
140           13698.000       1.621        1.156       6.000   34.84%       5.000
                  6.000
141      MSQTOT4                4.732       -1.233       0.000    1.17%       4.000
            5.000       5.000
142           10899.000       1.822        1.268       6.000   35.05%       5.000
                  6.000
143      MSQTOT7                4.569       -1.160       0.000    2.00%       4.000
            5.000       5.000
144             8169.000       2.033        1.060       6.000   30.31%       5.000
                  6.000
145
146 RANDOM STARTS RESULTS RANKED FROM THE BEST TO THE WORST LOGLIKELIHOOD VALUES
147
148 3 perturbed starting value run(s) did not converge in the initial stage
149 optimizations.
150
151 Final stage loglikelihood values at local maxima, seeds, and initial stage start
        numbers:
```

```
152
153            -50638.523  851945           18
154            -50638.523  573096           20
155            -50638.523  107446           12
156            -50638.523  285380           1
157
158
159
160 THE BEST LOGLIKELIHOOD VALUE HAS BEEN REPLICATED.  RERUN WITH AT LEAST TWICE THE
161 RANDOM STARTS TO CHECK THAT THE BEST LOGLIKELIHOOD IS STILL OBTAINED AND REPLICATED.
162
163
164 THE MODEL ESTIMATION TERMINATED NORMALLY
165
166
167
168 MODEL FIT INFORMATION
169
170 Number of Free Parameters                       12
171
172 Loglikelihood
173
174            H0 Value                       -50638.523
175            H0 Scaling Correction Factor      1.4852
176              for MLR
177
178 Information Criteria
179
180            Akaike (AIC)                   101301.047
181            Bayesian (BIC)                 101391.659
182            Sample-Size Adjusted BIC       101353.524
183              (n* = (n + 2) / 24)
```

```
184
185
186
187 FINAL CLASS COUNTS AND PROPORTIONS FOR THE LATENT CLASSES
188 BASED ON THE ESTIMATED MODEL
189
190     Latent
191    Classes
192
193        1      10336.24310          0.73520
194        2       3722.75690          0.26480
195
196
197 FINAL CLASS COUNTS AND PROPORTIONS FOR THE LATENT CLASSES
198 BASED ON ESTIMATED POSTERIOR PROBABILITIES
199
200     Latent
201    Classes
202
203        1      10336.24310          0.73520
204        2       3722.75690          0.26480
205
206
207 FINAL CLASS COUNTS AND PROPORTIONS FOR THE LATENT CLASSES
208 BASED ON THEIR MOST LIKELY LATENT CLASS MEMBERSHIP
209
210 Class Counts and Proportions
211
212     Latent
213    Classes
214
215        1            11061          0.78676
```

```
       2              2998          0.21324


CLASSIFICATION QUALITY

     Entropy                         0.629


Average Latent Class Probabilities for Most Likely Latent Class Membership (Row)
by Latent Class (Column)

           1        2

    1   0.904    0.096
    2   0.113    0.887


Classification Probabilities for the Most Likely Latent Class Membership (Column)
by Latent Class (Row)

           1        2

    1   0.967    0.033
    2   0.286    0.714


Logits for the Classification Probabilities for the Most Likely Latent Class Membership
        (Column)
by Latent Class (Row)

              1        2
```

```
247      1       3.386     0.000
248      2      -0.917     0.000
249
250
251 MODEL RESULTS
252
253                                                    Two-Tailed
254                     Estimate       S.E.  Est./S.E.    P-Value
255
256 Latent Class 1
257
258  I        |
259     MSQTOT1            1.000      0.000    999.000    999.000
260     MSQTOT4            1.000      0.000    999.000    999.000
261     MSQTOT7            1.000      0.000    999.000    999.000
262
263  S        |
264     MSQTOT1            0.000      0.000    999.000    999.000
265     MSQTOT4            0.600      0.000    999.000    999.000
266     MSQTOT7            1.200      0.000    999.000    999.000
267
268  S        WITH
269     I                 -0.050      0.027     -1.879      0.060
270
271  Means
272     I                  5.186      0.028    181.962      0.000
273     S                 -0.065      0.019     -3.430      0.001
274
275  Intercepts
276     MSQTOT1            0.000      0.000    999.000    999.000
277     MSQTOT4            0.000      0.000    999.000    999.000
278     MSQTOT7            0.000      0.000    999.000    999.000
```

```
279
280  Variances
281     I                  0.160      0.039      4.110      0.000
282     S                  0.021      0.059      0.363      0.716
283
284  Residual Variances
285     MSQTOT1            0.712      0.027     26.765      0.000
286     MSQTOT4            0.710      0.018     39.393      0.000
287     MSQTOT7            0.826      0.038     21.534      0.000
288
289 Latent Class 2
290
291  I        |
292     MSQTOT1            1.000      0.000    999.000    999.000
293     MSQTOT4            1.000      0.000    999.000    999.000
294     MSQTOT7            1.000      0.000    999.000    999.000
295
296  S        |
297     MSQTOT1            0.000      0.000    999.000    999.000
298     MSQTOT4            0.600      0.000    999.000    999.000
299     MSQTOT7            1.200      0.000    999.000    999.000
300
301  S        WITH
302     I                 -0.050      0.027     -1.879      0.060
303
304  Means
305     I                  3.692      0.033    112.290      0.000
306     S                 -1.181      0.122     -9.687      0.000
307
308  Intercepts
309     MSQTOT1            0.000      0.000    999.000    999.000
310     MSQTOT4            0.000      0.000    999.000    999.000
```

```
311     MSQTOT7            0.000      0.000    999.000    999.000
312
313  Variances
314     I                  1.391      0.059     23.684      0.000
315     S                  0.021      0.059      0.363      0.716
316
317  Residual Variances
318     MSQTOT1            0.712      0.027     26.765      0.000
319     MSQTOT4            0.710      0.018     39.393      0.000
320     MSQTOT7            0.826      0.038     21.534      0.000
321
322 Categorical Latent Variables
323
324  Means
325     C#1                1.021      0.080     12.784      0.000
326
327
328 QUALITY OF NUMERICAL RESULTS
329
330      Condition Number for the Information Matrix              0.593E-03
331        (ratio of smallest to largest eigenvalue)
332
333
334 TECHNICAL 1 OUTPUT
335
336
337      PARAMETER SPECIFICATION FOR LATENT CLASS 1
338
339
340            NU
341               MSQTOT1       MSQTOT4       MSQTOT7
342               --------      --------      --------
```

```
                    0             0             0

           LAMBDA
              I             S
              ________      ________
MSQTOT1             0             0
MSQTOT4             0             0
MSQTOT7             0             0

           THETA
              MSQTOT1       MSQTOT4       MSQTOT7
              ________      ________      ________
MSQTOT1             1
MSQTOT4             0             2
MSQTOT7             0             0             3

           ALPHA
              I             S
              ________      ________
                    4             5

           BETA
              I             S
              ________      ________
I                   0             0
S                   0             0
```

```
375             PSI
376                I              S
377                ________       ________
378 I                    6
379 S                    7              8
380
381
382      PARAMETER SPECIFICATION FOR LATENT CLASS 2
383
384
385             NU
386                MSQTOT1        MSQTOT4        MSQTOT7
387                ________       ________       ________
388                      0              0              0
389
390
391             LAMBDA
392                I              S
393                ________       ________
394 MSQTOT1              0              0
395 MSQTOT4              0              0
396 MSQTOT7              0              0
397
398
399             THETA
400                MSQTOT1        MSQTOT4        MSQTOT7
401                ________       ________       ________
402 MSQTOT1              1
403 MSQTOT4              0              2
404 MSQTOT7              0              0              3
405
406
```

```
407             ALPHA
408                I              S
409                --------       --------
410                      9             10
411
412
413             BETA
414                I              S
415                --------       --------
416  I                   0              0
417  S                   0              0
418
419
420             PSI
421                I              S
422                --------       --------
423  I                  11
424  S                   7              8
425
426
427       PARAMETER SPECIFICATION FOR LATENT CLASS REGRESSION MODEL PART
428
429
430             ALPHA(C)
431                C#1            C#2
432                --------       --------
433                     12              0
434
435
436       STARTING VALUES FOR LATENT CLASS 1
437
438
```

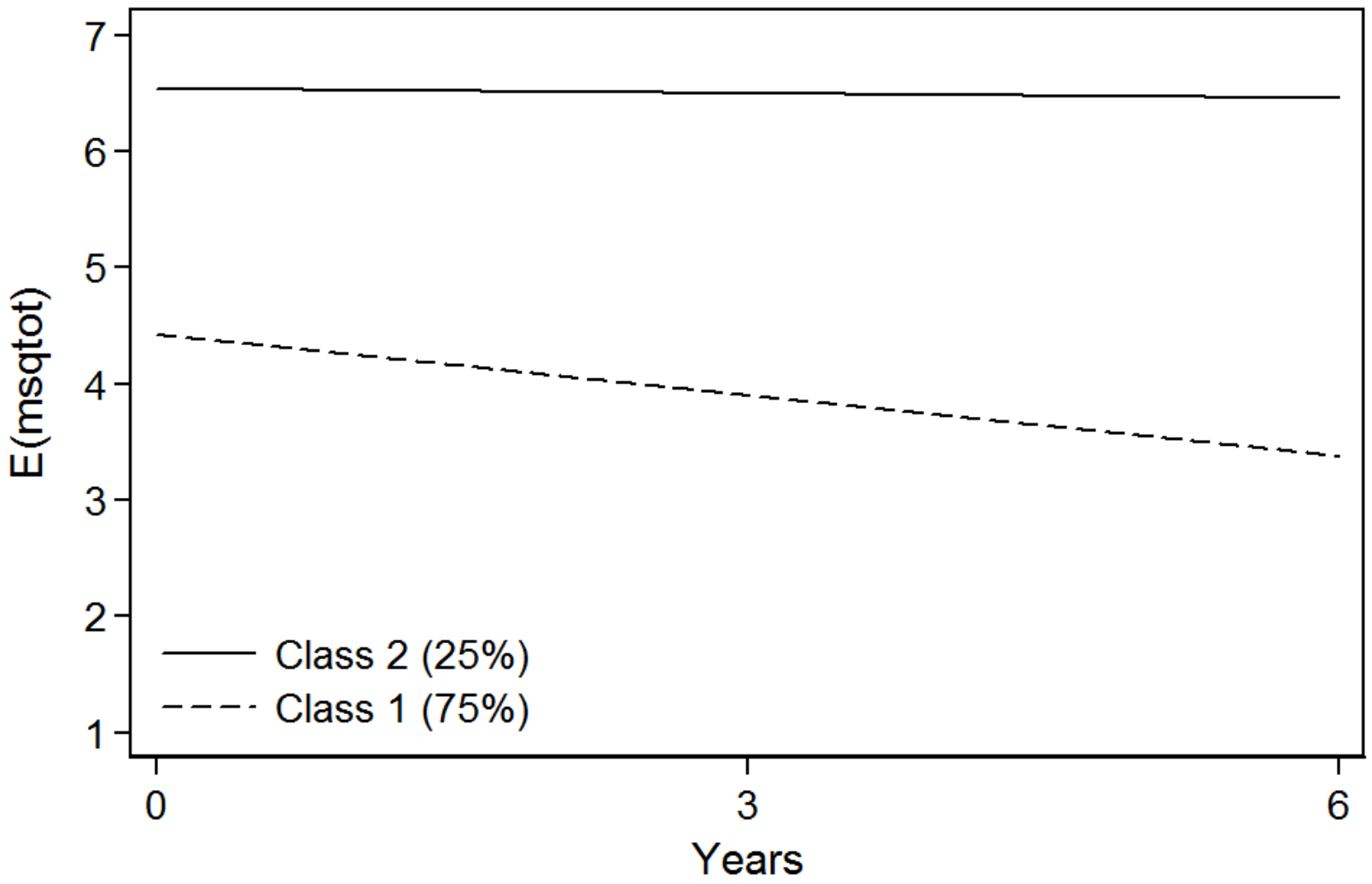
```
439             NU
440                MSQTOT1        MSQTOT4        MSQTOT7
441                --------       --------       --------
442                  0.000          0.000          0.000
443
444
445             LAMBDA
446                I              S
447                --------       --------
448  MSQTOT1         1.000          0.000
449  MSQTOT4         1.000          0.600
450  MSQTOT7         1.000          1.200
451
452
453             THETA
454                MSQTOT1        MSQTOT4        MSQTOT7
455                --------       --------       --------
456  MSQTOT1         0.810
457  MSQTOT4         0.000          0.911
458  MSQTOT7         0.000          0.000          1.016
459
460
461             ALPHA
462                I              S
463                --------       --------
464                  4.948         -0.406
465
466
467             BETA
468                I              S
469                --------       --------
470  I               0.000          0.000
```

```
S                0.000          0.000

           PSI
              I              S
              --------       --------
I               1.247
S               0.000          2.157


    STARTING VALUES FOR LATENT CLASS 2


           NU
              MSQTOT1        MSQTOT4        MSQTOT7
              --------       --------       --------
                0.000          0.000          0.000

           LAMBDA
              I              S
              --------       --------
MSQTOT1         1.000          0.000
MSQTOT4         1.000          0.600
MSQTOT7         1.000          1.200

           THETA
              MSQTOT1        MSQTOT4        MSQTOT7
              --------       --------       --------
MSQTOT1         0.810
MSQTOT4         0.000          0.911
```

```
MSQTOT7          0.000          0.000          1.016


           ALPHA
              I              S
              --------       --------
                4.948         -0.406


           BETA
              I              S
              --------       --------
I               0.000          0.000
S               0.000          0.000


           PSI
              I              S
              --------       --------
I               1.247
S               0.000          2.157


     STARTING VALUES FOR LATENT CLASS REGRESSION MODEL PART


           ALPHA(C)
              C#1            C#2
              --------       --------
                0.000          0.000
```

```
TECHNICAL 11 OUTPUT

     Random Starts Specifications for the k-1 Class Analysis Model
        Number of initial stage random starts                    20
        Number of final stage optimizations                       4



     VUONG-LO-MENDELL-RUBIN LIKELIHOOD RATIO TEST FOR 1 (H0) VERSUS 2 CLASSES

          H0 Loglikelihood Value                        -52691.657
          2 Times the Loglikelihood Difference            4106.268
          Difference in the Number of Parameters                 4
          Mean                                             35.303
          Standard Deviation                               52.154
          P-Value                                          0.0000

     LO-MENDELL-RUBIN ADJUSTED LRT TEST

          Value                                          4001.527
          P-Value                                          0.0000


TECHNICAL 14 OUTPUT

     Random Starts Specifications for the k-1 Class Analysis Model
        Number of initial stage random starts                    20
        Number of final stage optimizations                       4

     Random Starts Specification for the k-1 Class Model for Generated Data
        Number of initial stage random starts                     0
        Number of final stage optimizations for the
```

```
          initial stage random starts                    0
     Random Starts Specification for the k Class Model for Generated Data
        Number of initial stage random starts              40
        Number of final stage optimizations                 8
     Number of bootstrap draws requested                Varies


     PARAMETRIC BOOTSTRAPPED LIKELIHOOD RATIO TEST FOR 1 (H0) VERSUS 2 CLASSES

          H0 Loglikelihood Value                    -52691.657
          2 Times the Loglikelihood Difference        4106.268
          Difference in the Number of Parameters             4
          Approximate P-Value                           0.0000
          Successful Bootstrap Draws                         5

     WARNING:  OF THE 5 BOOTSTRAP DRAWS, 4 DRAWS HAD BOTH A SMALLER LRT VALUE THAN THE
     OBSERVED LRT VALUE AND NOT A REPLICATED BEST LOGLIKELIHOOD VALUE FOR THE 2-CLASS
          MODEL.
     THIS MEANS THAT THE P-VALUE MAY NOT BE TRUSTWORTHY DUE TO LOCAL MAXIMA.
     INCREASE THE NUMBER OF RANDOM STARTS USING THE LRTSTARTS OPTION.



     Beginning Time:  13:41:00
        Ending Time:  13:41:13
       Elapsed Time:  00:00:13


Mplus VERSION 8.8 DEMO (Mac) has the following limitations:
  Maximum number of dependent variables: 6
  Maximum number of independent variables: 2
  Maximum number of between variables: 2
```


7
6
5
4
3
2
1
E(msqtot)
Class 2 (25%)
Class 1 (75%)
0
3
6
Years

Mixture Model Fit

- Convergence & Best loglikelihood replication
- Deviance
- Entropy
- Posterior Class Probability
- LMVR

Pace of Cognitive Decline

Model	N	5y Change
Single Class	14,059	-0.468
Two Classes		
Class 1	10,580	-0.068
Class 2	3,479	-0.871

Warning

- Spurious classes (over extraction)
- Especially with non-normal outcomes (floor/ceiling effects)
- Bauer, D. J., & Curran, P. J. (2003). Distributional assumptions of growth mixture models: Implications for overextraction of latent trajectory classes. Psychological Methods, 8(3), 338-363.

Determining Classes

- Model Testing
  - LogLikelihood Tests (OUTPUT: Tech 11 Tech 14;)
  - Information criteria (AIC, BIC, saBIC)
- Quality of classification
  - Classification table based on posterior class probabilities
  - Entropy
- Interpretability and usefulness
  - Theory
  - Predictive validity
  - Exogenous variables

```
TECHNICAL 11 OUTPUT

     Random Starts Specifications for the k-1 Class Analysis Model
        Number of initial stage random starts                    20
        Number of final stage optimizations                       4



     VUONG-LO-MENDELL-RUBIN LIKELIHOOD RATIO TEST FOR 1 (H0) VERSUS 2 CLASSES

          H0 Loglikelihood Value                          -52702.406
          2 Times the Loglikelihood Difference              6699.310
          Difference in the Number of Parameters                   7
          Mean                                               157.712
          Standard Deviation                                 224.336
          P-Value                                              0.0000

     LO-MENDELL-RUBIN ADJUSTED LRT TEST

          Value                                             6600.583
          P-Value                                              0.0000
```

ex0206.out.tech11.txt

```
TECHNICAL 14 OUTPUT

     Random Starts Specifications for the k-1 Class Analysis Model
        Number of initial stage random starts                    20
        Number of final stage optimizations                       4

     Random Starts Specification for the k-1 Class Model for Generated Data
        Number of initial stage random starts                     0
        Number of final stage optimizations for the
           initial stage random starts                            0
     Random Starts Specification for the k Class Model for Generated Data
        Number of initial stage random starts                    40
        Number of final stage optimizations                       8
     Number of bootstrap draws requested                      Varies


     PARAMETRIC BOOTSTRAPPED LIKELIHOOD RATIO TEST FOR 1 (H0) VERSUS 2 CLASSES

          H0 Loglikelihood Value                         -52702.406
          2 Times the Loglikelihood Difference            6699.310
          Difference in the Number of Parameters                 7
          Approximate P-Value                               0.0000
          Successful Bootstrap Draws                             5
```

ex0206.out.tech14.txt

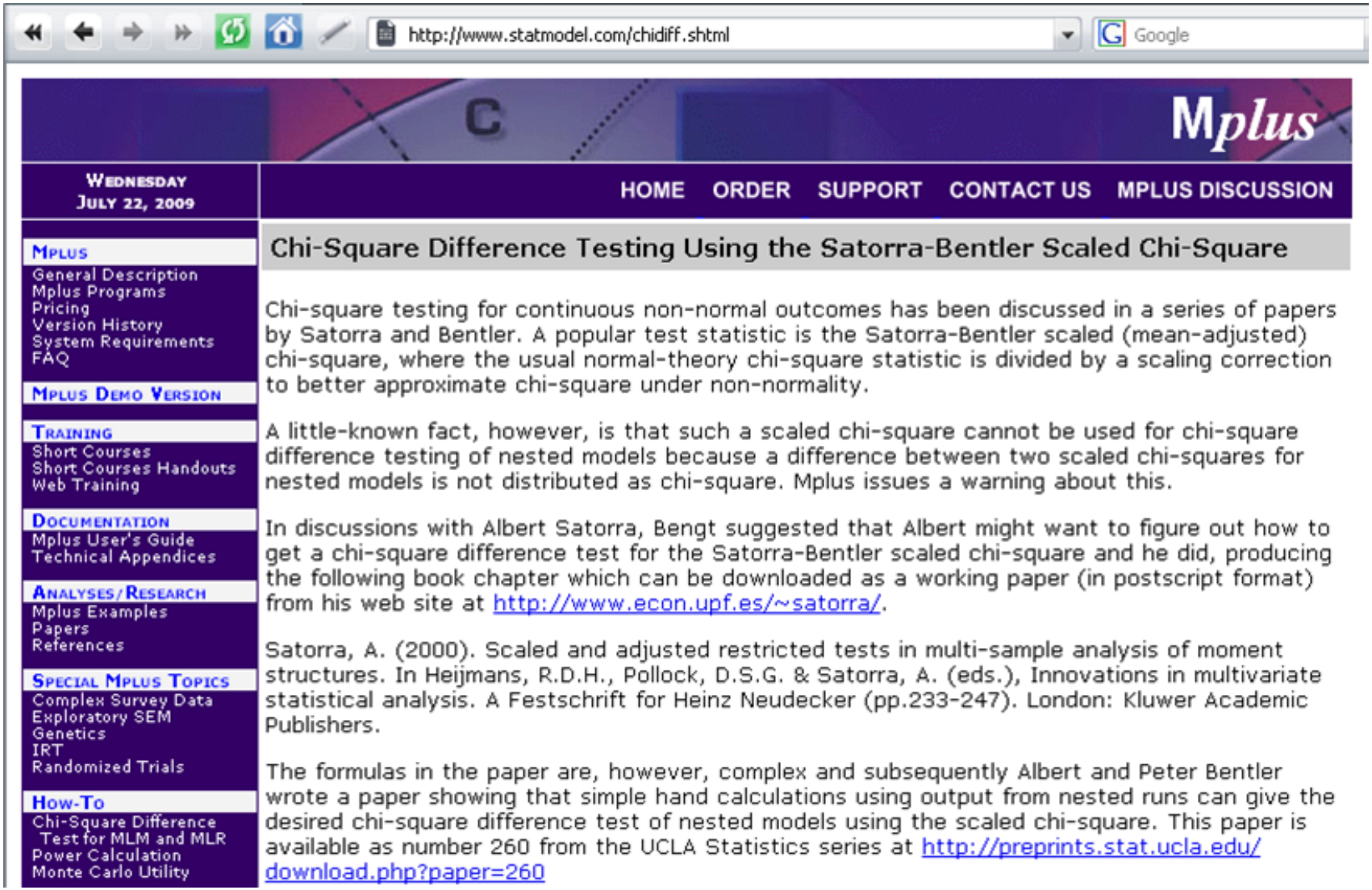
```
TITLE:    Growth mixture model, MSQ sum scores
DATA:     FILE = ex0201.dat;
VARIABLE: NAMES = msqtot1 msqtot4 msqtot7 ;
          MISSING  = ALL (-9999) ;
          CLASSES = c(1) ; ! name class var & define # of levels
OUTPUT:   TECH1 ;
       TECH14;
          TECH11 ;
ANALYSIS: TYPE = MIXTURE ;  ! ask for a mixture model
MODEL:    %OVERALL%
          i s | msqtot1@0 msqtot4@.6 msqtot7@1.2 ;
          %c#1%
          msqtot1-msqtot7 (1) ;
          [i ];
          [s ] (a1) ;
          i s; i with s ;
          !%c#2%
          !msqtot1-msqtot7 (2) ;
          ![i ];
          ![s ] (a2) ;
          ! i s; i with s ;
```

ex0206c1.inp

Number of Classes	1	2
Loglikelihood (H0)	-52702.406	-50638.523
LL scaling factor	1.3320	1.4852
Num param	6	12
AIC	105416.813	101301.047
BIC	105462.119	101391.659
saBIC	105443.051	101353.524
Entropy	na	0.629
Satorra-Bentler adjusted LRT	na	$P = 1.000$
TECH 11 Lo-Mendel-Rubin adjusted LRT	na	$P < 0.001$
TECH 14 McLachlan-Peel bootstrap LRT	na	$P < 0.001$

Loglikelihood

- H0: analysis model, estimated model
- H1: unrestricted or less restricted model
- H0 is nested in H1
- Higher is better
- $-2\times$ difference in LL for model comparison
- Use Satorra-Bentler scaling adjustment for MLR, WLSMV estimator (see here)


http://www.statmodel.com/chidiff.shtml
Google
Mplus
WEDNESDAY
JULY 22, 2009
HOME ORDER SUPPORT CONTACT US MPLUS DISCUSSION
MPLUS
General Description
Mplus Programs
Pricing
Version History
System Requirements
FAQ
MPLUS DEMO VERSION
TRAINING
Short Courses
Short Courses Handouts
Web Training
DOCUMENTATION
Mplus User's Guide
Technical Appendices
ANALYSES/RESEARCH
Mplus Examples
Papers
References
SPECIAL MPLUS TOPICS
Complex Survey Data
Exploratory SEM
Genetics
IRT
Randomized Trials
HOW-TO
Chi-Square Difference Test for MLM and MLR
Power Calculation
Monte Carlo Utility
Chi-Square Difference Testing Using the Satorra-Bentler Scaled Chi-Square
Chi-square testing for continuous non-normal outcomes has been discussed in a series of papers by Satorra and Bentler. A popular test statistic is the Satorra-Bentler scaled (mean-adjusted) chi-square, where the usual normal-theory chi-square statistic is divided by a scaling correction to better approximate chi-square under non-normality.
A little-known fact, however, is that such a scaled chi-square cannot be used for chi-square difference testing of nested models because a difference between two scaled chi-squares for nested models is not distributed as chi-square. Mplus issues a warning about this.
In discussions with Albert Satorra, Bengt suggested that Albert might want to figure out how to get a chi-square difference test for the Satorra-Bentler scaled chi-square and he did, producing the following book chapter which can be downloaded as a working paper (in postscript format) from his web site at http://www.econ.upf.es/~satorra/.
Satorra, A. (2000). Scaled and adjusted restricted tests in multi-sample analysis of moment structures. In Heijmans, R.D.H., Pollock, D.S.G. & Satorra, A. (eds.), Innovations in multivariate statistical analysis. A Festschrift for Heinz Neudecker (pp.233-247). London: Kluwer Academic Publishers.
The formulas in the paper are, however, complex and subsequently Albert and Peter Bentler wrote a paper showing that simple hand calculations using output from nested runs can give the desired chi-square difference test of nested models using the scaled chi-square. This paper is available as number 260 from the UCLA Statistics series at http://preprints.stat.ucla.edu/download.php?paper=260

Difference Testing Using the Loglikelihood

Following are the steps needed to compute a chi-square difference test based on loglikelihood values and scaling correction factors obtained with the MLR estimator.

1. Estimate the nested and comparison models using MLR. The printout gives loglikelihood values L0 and L1 for the H0 and H1 models, respectively, as well as scaling correction factors c0 and c1 for the H0 and H1 models, respectively. For example,

```
L0 = -2,606, c0 = 1.450 with 39 parameters (p0 = 39)
L1 = -2,583, c1 = 1.546 with 47 parameters (p1 = 47)
```

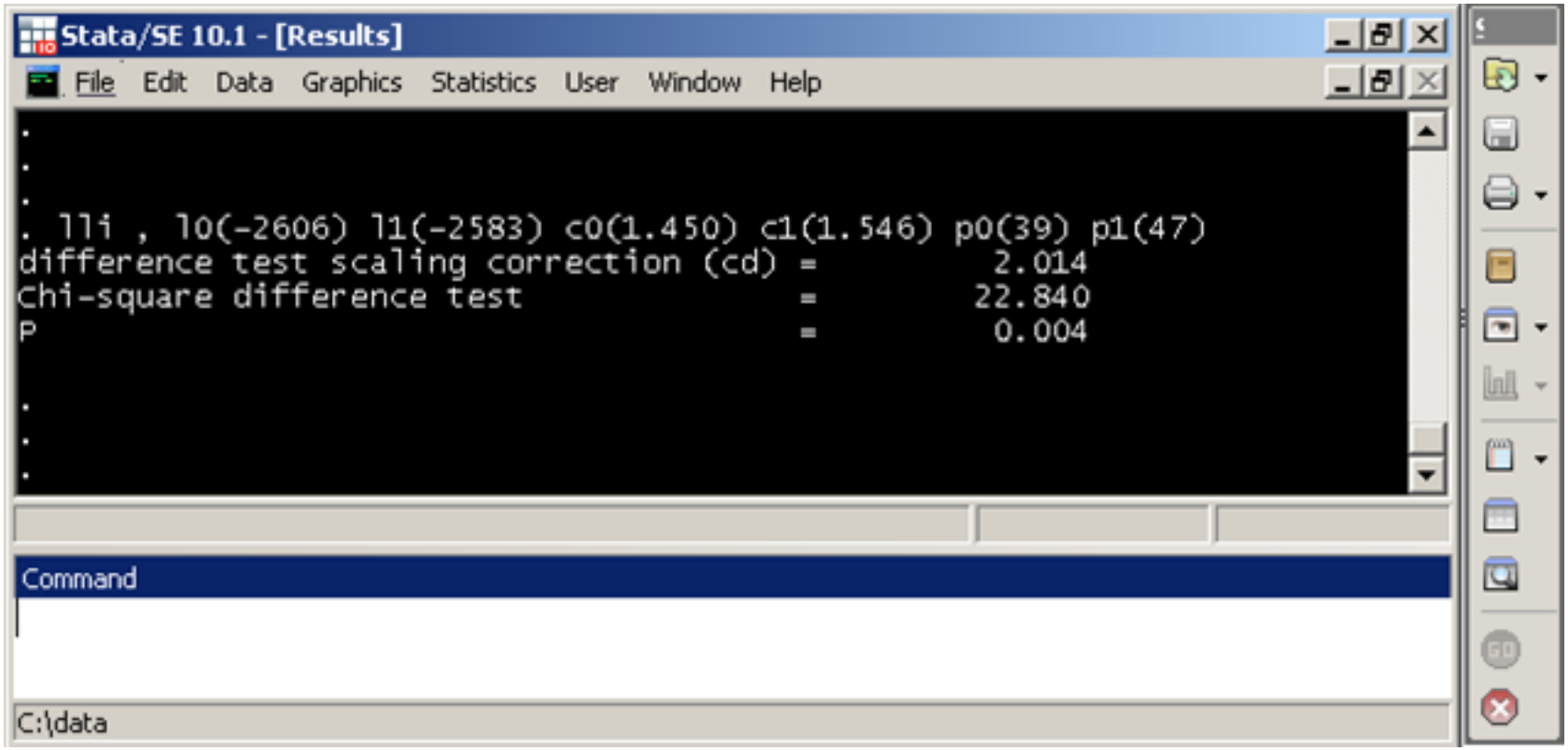
2. Compute the difference test scaling correction where p0 is the number of parameters in the nested model and p1 is the number of parameters in the comparison model.

```
cd = (p0 * c0 - p1*c1)/(p0 - p1)
   = (39*1.450 - 47*1.546)/(39 - 47) = 2.014
```

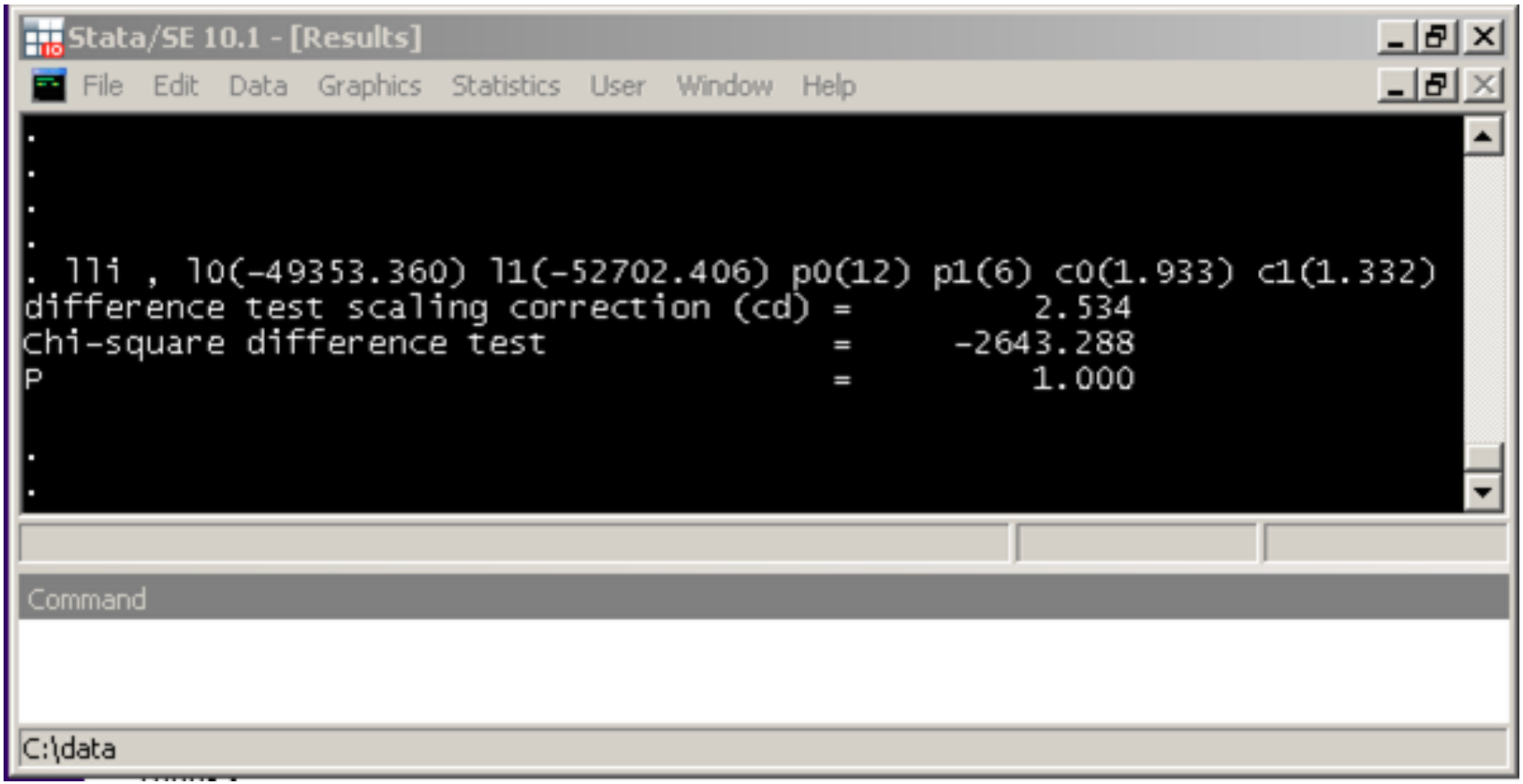
3. Compute the chi-square difference test (TRd) as follows:

```
TRd = -2*(L0 - L1)/cd
    = -2*(-2606 + 2583)/2.014 = 22.840
```

Figure it out once and then program it into an Excel spreadsheet, SAS macro, or Stata ado (e.g., or get lli.ado from me!)


Stata/SE 10.1 - [Results]
File Edit Data Graphics Statistics User Window Help
. lli , l0(-2606) l1(-2583) c0(1.450) c1(1.546) p0(39) p1(47)
difference test scaling correction (cd) = 2.014
Chi-square difference test = 22.840
P = 0.004
Command
C:\data

The asymptotics of this correction does not always work and seemingly inadmissible values are returned. This is discussed in an Mplus Discussion thread by Bength Muthén (here)



Lo-Mendell-Rubin LRT (Tech 11)

- Lo, Mendell, Rubin (2001). Testing the number of components in a normal mixture. Biometrika, 88, 767-78
- Compares the estimated model (H0) with a less-restricted (one less class) model (H1)
- P-value is the probability that the data have been generated by the model with one less class
- Low P-value: the H1 model is rejected in favor of the H0 model
- An adjustment to a test developed by Vuong, Q. H. (1989). Likelihood ratio tests for model selection and non-nested hypotheses. Econometrica, 57, 307-333.

Parametric Bootstrapped LRT (Tech 14)

- McLachlan and Peel (2000). Finite Mixture Models. Wiley.
- Compares the estimated model (H0) with a less-restricted (one less class) model (H1)
- P-value is an approximation of the probability that the data have been generated by the model with one less class
- Low P-value: the H1 model is rejected in favor of the H0 model

Information Criteria

$$\text{Akaike, } AIC = -2 \times LL + 2r$$
$$\text{Bayesian, } BIC = -2 \times LL + r + ln(n)$$
$$\text{Sample Size Adjusted, } saBIC = -2 \times LL + r + ln([n + 2]/24)$$

LL, Loglikelihood

r, number of free parameters

n, number of observations

For all, lower is better. Arbitrary scaling. Useful for comparing non-nested models based on the same data. Not all authors (and software packages) use the same formula for the IC, but all are based on LL and r.

Entropy

- Entropy with values approaching 1 indicate clear delineation of classes
  - Celeux, G., & Soromenho, G. (1996). An entropy criterion for assessing the number of clusters in a mixture model. Journal of Classification, 13, 195-212.
- *You really want an entropy of 0.8 or higher for a model to be able to clearly identify individuals following different trajectory types (although lower entropies can still produce good parameter estimates).*
  - Bengt O. Muthén, Mplus Discussion

Growth Mixture Model With Covariates

ex0207.inp

```
TITLE:     Growth mixture model, msq sum scores from EPESE
DATA:      FILE = ex0201.dat;
VARIABLE:  NAMES = msqtot1 msqtot4 msqtot7 age black ;
           MISSING ARE ALL (-9999) ;
           CLASSES = c(2)  ;
DEFINE:    CENTER age black (GRANDMEAN) ;
OUTPUT:    TECH1  TECH11 TECH14 ;
ANALYSIS:  TYPE = mixture  ;
MODEL:     %overall%
           i s | msqtot1@0 msqtot4@.6 msqtot7@1.2 ;
           c#1 on age black  ;
           %c#1%
           msqtot1-msqtot7* (1) ;
           [i* s*] ;
           i* s* ;
           %c#2%
           msqtot1-msqtot7* (2) ;
           [i* s*] ;
           i* s* ;
```

```
Mplus VERSION 8.8 DEMO (Mac)
MUTHEN & MUTHEN
07/11/2022   1:41 PM

INPUT INSTRUCTIONS

  TITLE:     Growth mixture model, msq sum scores from EPESE
  DATA:      FILE = ex0201.dat;
  VARIABLE:  NAMES = msqtot1 msqtot4 msqtot7 age black ;
             MISSING ARE ALL (-9999) ;
             CLASSES = c(2)  ;
  DEFINE:    CENTER age black (GRANDMEAN) ;
  OUTPUT:    TECH1  TECH11 TECH14 ;
  ANALYSIS:  TYPE = mixture  ;
  MODEL:     %overall%
             i s | msqtot1@0 msqtot4@.6 msqtot7@1.2 ;
             c#1 on age black  ;
             %c#1%
             msqtot1-msqtot7* (1) ;
             [i* s*] ;
             i* s* ;
             %c#2%
             msqtot1-msqtot7* (2) ;
             [i* s*] ;
             i* s* ;



*** WARNING
  Data set contains cases with missing on all variables except
  x-variables.  These cases were not included in the analysis.
  Number of cases with missing on all variables except x-variables:  397
```

```
   1 WARNING(S) FOUND IN THE INPUT INSTRUCTIONS


Growth mixture model, msq sum scores from EPESE

SUMMARY OF ANALYSIS

Number of groups                                                 1
Number of observations                                       14059

Number of dependent variables                                    3
Number of independent variables                                  2
Number of continuous latent variables                            2
Number of categorical latent variables                           1

Observed dependent variables

  Continuous
   MSQTOT1     MSQTOT4     MSQTOT7

Observed independent variables
   AGE         BLACK

Continuous latent variables
   I           S

Categorical latent variables
   C

Variables with special functions

```

```
  Centering (GRANDMEAN)
    AGE          BLACK


Estimator                                                      MLR
Information matrix                                        OBSERVED
Optimization Specifications for the Quasi-Newton Algorithm for
Continuous Outcomes
  Maximum number of iterations                                 100
  Convergence criterion                                0.100D-05
Optimization Specifications for the EM Algorithm
  Maximum number of iterations                                 500
  Convergence criteria
    Loglikelihood change                               0.100D-06
    Relative loglikelihood change                      0.100D-06
    Derivative                                         0.100D-05
Optimization Specifications for the M step of the EM Algorithm for
Categorical Latent variables
  Number of M step iterations                                    1
  M step convergence criterion                         0.100D-05
  Basis for M step termination                         ITERATION
Optimization Specifications for the M step of the EM Algorithm for
Censored, Binary or Ordered Categorical (Ordinal), Unordered
Categorical (Nominal) and Count Outcomes
  Number of M step iterations                                    1
  M step convergence criterion                         0.100D-05
  Basis for M step termination                         ITERATION
  Maximum value for logit thresholds                          15
  Minimum value for logit thresholds                         -15
  Minimum expected cell size for chi-square            0.100D-01
Maximum number of iterations for H1                         2000
Convergence criterion for H1                           0.100D-03
```

```
Optimization algorithm                                          EMA
Random Starts Specifications
  Number of initial stage random starts                          20
  Number of final stage optimizations                             4
  Number of initial stage iterations                             10
  Initial stage convergence criterion                     0.100D+01
  Random starts scale                                     0.500D+01
  Random seed for generating random starts                        0

Input data file(s)
  ex0201.dat
Input data format  FREE


SUMMARY OF DATA

     Number of missing data patterns             7
     Number of y missing data patterns           7
     Number of u missing data patterns           0


COVARIANCE COVERAGE OF DATA

Minimum covariance coverage value   0.100


     PROPORTION OF DATA PRESENT FOR Y


           Covariance Coverage
              MSQTOT1       MSQTOT4       MSQTOT7       AGE           BLACK
              --------      --------      --------      --------      --------
```

```
 MSQTOT1        0.974
 MSQTOT4        0.752          0.775
 MSQTOT7        0.565          0.561          0.581
 AGE            0.974          0.775          0.581          1.000
 BLACK          0.974          0.775          0.581          1.000          1.000


UNIVARIATE SAMPLE STATISTICS


    UNIVARIATE HIGHER-ORDER MOMENT DESCRIPTIVE STATISTICS

        Variable/          Mean/     Skewness/   Minimum/ % with
                Percentiles
           Sample Size       Variance    Kurtosis    Maximum  Min/Max      20%/60%    40%/
               80%    Median

    MSQTOT1                4.768      -1.152       0.000    0.70%       4.000
          5.000       5.000
           13698.000       1.621       1.156       6.000   34.84%       5.000
                6.000
    MSQTOT4                4.732      -1.233       0.000    1.17%       4.000
          5.000       5.000
           10899.000       1.822       1.268       6.000   35.05%       5.000
                6.000
    MSQTOT7                4.569      -1.160       0.000    2.00%       4.000
          5.000       5.000
            8169.000       2.033       1.060       6.000   30.31%       5.000
                6.000
    AGE                    0.000       0.583      -6.932   31.36%      -6.932
          -1.932      -1.932
```

```
152            14059.000      40.275      -0.736      13.068     8.17%       3.068
                    8.068
153      BLACK                  0.000       1.528      -0.196    80.35%      -0.196
              -0.196     -0.196
154            14059.000       0.158       0.335       0.804    19.65%      -0.196
                   -0.196
155
156 RANDOM STARTS RESULTS RANKED FROM THE BEST TO THE WORST LOGLIKELIHOOD VALUES
157
158 Final stage loglikelihood values at local maxima, seeds, and initial stage start
        numbers:
159
160           -48497.170  107446           12
161           -48497.170  903420           5
162           -48497.170  939021           8
163
164 1 perturbed starting value run(s) did not converge.
165
166
167
168 THE BEST LOGLIKELIHOOD VALUE HAS BEEN REPLICATED.  RERUN WITH AT LEAST TWICE THE
169 RANDOM STARTS TO CHECK THAT THE BEST LOGLIKELIHOOD IS STILL OBTAINED AND REPLICATED.
170
171
172 THE MODEL ESTIMATION TERMINATED NORMALLY
173
174
175
176 MODEL FIT INFORMATION
177
178 Number of Free Parameters                       14
179
```

```
Loglikelihood

          H0 Value                          -48497.170
          H0 Scaling Correction Factor           1.1125
            for MLR

Information Criteria

          Akaike (AIC)                       97022.341
          Bayesian (BIC)                     97128.055
          Sample-Size Adjusted BIC           97083.564
            (n* = (n + 2) / 24)



FINAL CLASS COUNTS AND PROPORTIONS FOR THE LATENT CLASSES
BASED ON THE ESTIMATED MODEL

    Latent
   Classes

       1       5843.15250          0.41562
       2       8215.84750          0.58438


FINAL CLASS COUNTS AND PROPORTIONS FOR THE LATENT CLASSES
BASED ON ESTIMATED POSTERIOR PROBABILITIES

    Latent
   Classes

       1       5843.15253          0.41562
```

```
212        2       8215.84747          0.58438
213
214
215 FINAL CLASS COUNTS AND PROPORTIONS FOR THE LATENT CLASSES
216 BASED ON THEIR MOST LIKELY LATENT CLASS MEMBERSHIP
217
218 Class Counts and Proportions
219
220     Latent
221    Classes
222
223        1             5328          0.37897
224        2             8731          0.62103
225
226
227 CLASSIFICATION QUALITY
228
229      Entropy                         0.621
230
231
232 Average Latent Class Probabilities for Most Likely Latent Class Membership (Row)
233 by Latent Class (Column)
234
235             1        2
236
237     1   0.902    0.098
238     2   0.119    0.881
239
240
241 Classification Probabilities for the Most Likely Latent Class Membership (Column)
242 by Latent Class (Row)
243
```

```
244              1         2
245
246      1    0.823     0.177
247      2    0.063     0.937
248
249
250 Logits for the Classification Probabilities for the Most Likely Latent Class Membership
          (Column)
251 by Latent Class (Row)
252
253                1         2
254
255      1       1.534     0.000
256      2      -2.691     0.000
257
258
259 MODEL RESULTS
260
261                                                    Two-Tailed
262                     Estimate       S.E.  Est./S.E.    P-Value
263
264 Latent Class 1
265
266  I         |
267     MSQTOT1              1.000      0.000    999.000    999.000
268     MSQTOT4              1.000      0.000    999.000    999.000
269     MSQTOT7              1.000      0.000    999.000    999.000
270
271  S         |
272     MSQTOT1              0.000      0.000    999.000    999.000
273     MSQTOT4              0.600      0.000    999.000    999.000
274     MSQTOT7              1.200      0.000    999.000    999.000
```

```
275
276  S          WITH
277     I                 -0.006       0.012      -0.551       0.582
278
279  Means
280     I                  3.932       0.029     134.413       0.000
281     S                 -0.591       0.035     -17.018       0.000
282
283  Intercepts
284     MSQTOT1            0.000       0.000     999.000     999.000
285     MSQTOT4            0.000       0.000     999.000     999.000
286     MSQTOT7            0.000       0.000     999.000     999.000
287
288  Variances
289     I                  0.628       0.037      17.133       0.000
290     S                  0.439       0.048       9.108       0.000
291
292  Residual Variances
293     MSQTOT1            1.385       0.033      41.668       0.000
294     MSQTOT4            1.385       0.033      41.668       0.000
295     MSQTOT7            1.385       0.033      41.668       0.000
296
297 Latent Class 2
298
299  I          |
300     MSQTOT1            1.000       0.000     999.000     999.000
301     MSQTOT4            1.000       0.000     999.000     999.000
302     MSQTOT7            1.000       0.000     999.000     999.000
303
304  S          |
305     MSQTOT1            0.000       0.000     999.000     999.000
306     MSQTOT4            0.600       0.000     999.000     999.000
```

```
307     MSQTOT7            1.200      0.000    999.000    999.000
308
309  S        WITH
310     I                 -0.006      0.012     -0.551      0.582
311
312  Means
313     I                  5.404      0.019    285.510      0.000
314     S                 -0.143      0.015     -9.369      0.000
315
316  Intercepts
317     MSQTOT1            0.000      0.000    999.000    999.000
318     MSQTOT4            0.000      0.000    999.000    999.000
319     MSQTOT7            0.000      0.000    999.000    999.000
320
321  Variances
322     I                  0.092      0.013      7.120      0.000
323     S                  0.092      0.017      5.259      0.000
324
325  Residual Variances
326     MSQTOT1            0.389      0.012     31.377      0.000
327     MSQTOT4            0.389      0.012     31.377      0.000
328     MSQTOT7            0.389      0.012     31.377      0.000
329
330 Categorical Latent Variables
331
332  C#1        ON
333     AGE                0.129      0.005     27.124      0.000
334     BLACK              1.620      0.067     24.292      0.000
335
336  Intercepts
337     C#1               -0.384      0.063     -6.077      0.000
338
```

```
339
340 QUALITY OF NUMERICAL RESULTS
341
342      Condition Number for the Information Matrix              0.883E-04
343        (ratio of smallest to largest eigenvalue)
344
345
346 LOGISTIC REGRESSION ODDS RATIO RESULTS
347
348                                                  95% C.I.
349                     Estimate       S.E.  Lower 2.5% Upper 2.5%
350
351 Categorical Latent Variables
352
353  C#1       ON
354     AGE                 1.138      0.005      1.128      1.149
355     BLACK               5.051      0.337      4.432      5.756
356
357
358 ALTERNATIVE PARAMETERIZATIONS FOR THE CATEGORICAL LATENT VARIABLE REGRESSION
359
360                                                     Two-Tailed
361                     Estimate       S.E.  Est./S.E.    P-Value
362
363 Parameterization using Reference Class 1
364
365  C#2       ON
366     AGE                -0.129      0.005    -27.124      0.000
367     BLACK              -1.620      0.067    -24.292      0.000
368
369  Intercepts
370     C#2                 0.384      0.063      6.077      0.000
```

```
371
372
373 ODDS RATIO FOR THE ALTERNATIVE PARAMETERIZATIONS FOR THE CATEGORICAL LATENT VARIABLE
       REGRESSION
374
375                                                    95% C.I.
376                     Estimate       S.E.  Lower 2.5% Upper 2.5%
377
378 Parameterization using Reference Class 1
379
380  C#2      ON
381     AGE                 0.879      0.004      0.870      0.887
382     BLACK               0.198      0.013      0.174      0.226
383
384
385 TECHNICAL 1 OUTPUT
386
387
388      PARAMETER SPECIFICATION FOR LATENT CLASS 1
389
390
391            NU
392               MSQTOT1       MSQTOT4       MSQTOT7       AGE           BLACK
393               --------      --------      --------      --------      --------
394                     0             0             0             0             0
395
396
397            LAMBDA
398               I             S             AGE           BLACK
399               --------      --------      --------      --------
400  MSQTOT1            0             0             0             0
401  MSQTOT4            0             0             0             0
```

```
 MSQTOT7            0              0              0              0
 AGE                0              0              0              0
 BLACK              0              0              0              0


            THETA
               MSQTOT1        MSQTOT4        MSQTOT7        AGE            BLACK
               --------       --------       --------       --------       --------
 MSQTOT1             1
 MSQTOT4             0              1
 MSQTOT7             0              0              1
 AGE                 0              0              0              0
 BLACK               0              0              0              0              0


            ALPHA
               I              S              AGE            BLACK
               --------       --------       --------       --------
                     2              3              0              0


            BETA
               I              S              AGE            BLACK
               --------       --------       --------       --------
 I                   0              0              0              0
 S                   0              0              0              0
 AGE                 0              0              0              0
 BLACK               0              0              0              0


            PSI
               I              S              AGE            BLACK
```

```
                --------      --------      --------      --------
I                     4
S                     5             6
AGE                   0             0             0
BLACK                 0             0             0             0


    PARAMETER SPECIFICATION FOR LATENT CLASS 2


          NU
             MSQTOT1       MSQTOT4       MSQTOT7       AGE           BLACK
             --------      --------      --------      --------      --------
                   0             0             0             0             0


          LAMBDA
             I             S             AGE           BLACK
             --------      --------      --------      --------
MSQTOT1              0             0             0             0
MSQTOT4              0             0             0             0
MSQTOT7              0             0             0             0
AGE                  0             0             0             0
BLACK                0             0             0             0


          THETA
             MSQTOT1       MSQTOT4       MSQTOT7       AGE           BLACK
             --------      --------      --------      --------      --------
MSQTOT1               7
MSQTOT4               0             7
MSQTOT7               0             0             7
```

```
AGE                0              0              0              0
BLACK              0              0              0              0              0


           ALPHA
              I              S              AGE            BLACK
              --------       --------       --------       --------
                    8              9              0              0


           BETA
              I              S              AGE            BLACK
              --------       --------       --------       --------
I                   0              0              0              0
S                   0              0              0              0
AGE                 0              0              0              0
BLACK               0              0              0              0


           PSI
              I              S              AGE            BLACK
              --------       --------       --------       --------
I                  10
S                   5             11
AGE                 0              0              0
BLACK               0              0              0              0


     PARAMETER SPECIFICATION FOR LATENT CLASS REGRESSION MODEL PART


           ALPHA(C)
```

```
                C#1             C#2
                --------        --------
                     12               0


           GAMMA(C)
                AGE             BLACK
                --------        --------
C#1                  13              14
C#2                   0               0


     STARTING VALUES FOR LATENT CLASS 1


           NU
                MSQTOT1         MSQTOT4         MSQTOT7         AGE             BLACK
                --------        --------        --------        --------        --------
                  0.000           0.000           0.000           0.000           0.000


           LAMBDA
                I               S               AGE             BLACK
                --------        --------        --------        --------
MSQTOT1           1.000           0.000           0.000           0.000
MSQTOT4           1.000           0.600           0.000           0.000
MSQTOT7           1.000           1.200           0.000           0.000
AGE               0.000           0.000           1.000           0.000
BLACK             0.000           0.000           0.000           1.000


           THETA
```

```
                MSQTOT1       MSQTOT4       MSQTOT7       AGE           BLACK
                --------      --------      --------      --------      --------
MSQTOT1           0.810
MSQTOT4           0.000         0.911
MSQTOT7           0.000         0.000         1.016
AGE               0.000         0.000         0.000         0.000
BLACK             0.000         0.000         0.000         0.000         0.000


           ALPHA
              I             S             AGE           BLACK
              --------      --------      --------      --------
                4.948        -0.406         0.000         0.000


           BETA
              I             S             AGE           BLACK
              --------      --------      --------      --------
I                 0.000         0.000         0.000         0.000
S                 0.000         0.000         0.000         0.000
AGE               0.000         0.000         0.000         0.000
BLACK             0.000         0.000         0.000         0.000


           PSI
              I             S             AGE           BLACK
              --------      --------      --------      --------
I                 1.247
S                 0.000         2.157
AGE               0.000         0.000        20.138
BLACK             0.000         0.000         0.000         0.079
```

```
562
563       STARTING VALUES FOR LATENT CLASS 2
564
565
566             NU
567                MSQTOT1       MSQTOT4       MSQTOT7       AGE           BLACK
568                --------      --------      --------      --------      --------
569                  0.000         0.000         0.000         0.000         0.000
570
571
572             LAMBDA
573                I             S             AGE           BLACK
574                --------      --------      --------      --------
575   MSQTOT1        1.000         0.000         0.000         0.000
576   MSQTOT4        1.000         0.600         0.000         0.000
577   MSQTOT7        1.000         1.200         0.000         0.000
578   AGE            0.000         0.000         1.000         0.000
579   BLACK          0.000         0.000         0.000         1.000
580
581
582             THETA
583                MSQTOT1       MSQTOT4       MSQTOT7       AGE           BLACK
584                --------      --------      --------      --------      --------
585   MSQTOT1        0.810
586   MSQTOT4        0.000         0.911
587   MSQTOT7        0.000         0.000         1.016
588   AGE            0.000         0.000         0.000         0.000
589   BLACK          0.000         0.000         0.000         0.000         0.000
590
591
592             ALPHA
593                I             S             AGE           BLACK
```

```
              --------      --------      --------      --------
                4.948        -0.406         0.000         0.000


           BETA
              I             S             AGE           BLACK

              --------      --------      --------      --------
I               0.000         0.000         0.000         0.000
S               0.000         0.000         0.000         0.000
AGE             0.000         0.000         0.000         0.000
BLACK           0.000         0.000         0.000         0.000


           PSI
              I             S             AGE           BLACK

              --------      --------      --------      --------
I               1.247
S               0.000         2.157
AGE             0.000         0.000        20.138
BLACK           0.000         0.000         0.000         0.079


     STARTING VALUES FOR LATENT CLASS REGRESSION MODEL PART


           ALPHA(C)
              C#1           C#2

              --------      --------
                0.000         0.000


           GAMMA(C)
```

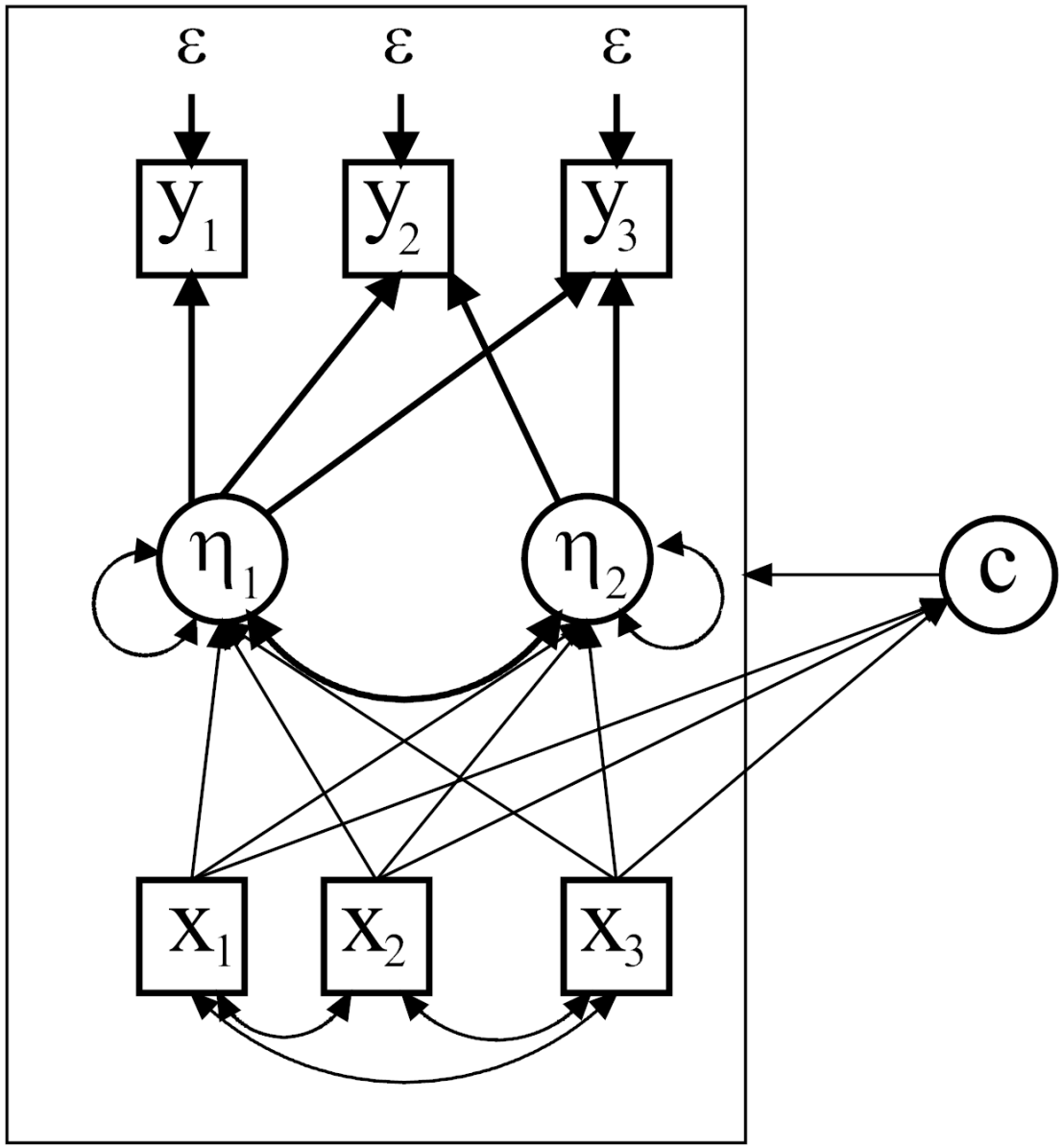
```
626                AGE             BLACK
627                --------        --------
628  C#1             0.000           0.000
629  C#2             0.000           0.000
630
631
632 TECHNICAL 11 OUTPUT
633
634      Random Starts Specifications for the k-1 Class Analysis Model
635         Number of initial stage random starts                    20
636         Number of final stage optimizations                       4
637
638
639
640      VUONG-LO-MENDELL-RUBIN LIKELIHOOD RATIO TEST FOR 1 (H0) VERSUS 2 CLASSES
641
642           H0 Loglikelihood Value                         -52702.406
643           2 Times the Loglikelihood Difference             8410.472
644           Difference in the Number of Parameters                  8
645           Mean                                              12.749
646           Standard Deviation                                20.538
647           P-Value                                           0.0000
648
649      LO-MENDELL-RUBIN ADJUSTED LRT TEST
650
651           Value                                           8301.821
652           P-Value                                           0.0000
653
654
655 TECHNICAL 14 OUTPUT
656
657      Random Starts Specifications for the k-1 Class Analysis Model
```

```
658         Number of initial stage random starts                    20
659         Number of final stage optimizations                       4
660
661       Random Starts Specification for the k-1 Class Model for Generated Data
662         Number of initial stage random starts                     0
663         Number of final stage optimizations for the
664            initial stage random starts                            0
665       Random Starts Specification for the k Class Model for Generated Data
666         Number of initial stage random starts                    40
667         Number of final stage optimizations                       8
668       Number of bootstrap draws requested                    Varies
669
670
671       PARAMETRIC BOOTSTRAPPED LIKELIHOOD RATIO TEST FOR 1 (H0) VERSUS 2 CLASSES
672
673            H0 Loglikelihood Value                         -52702.406
674            2 Times the Loglikelihood Difference             8410.472
675            Difference in the Number of Parameters                8
676            Approximate P-Value                              0.0000
677            Successful Bootstrap Draws                            5
678
679       WARNING:  OF THE 5 BOOTSTRAP DRAWS, 4 DRAWS HAD BOTH A SMALLER LRT VALUE THAN THE
680       OBSERVED LRT VALUE AND NOT A REPLICATED BEST LOGLIKELIHOOD VALUE FOR THE 2-CLASS
               MODEL.
681       THIS MEANS THAT THE P-VALUE MAY NOT BE TRUSTWORTHY DUE TO LOCAL MAXIMA.
682       INCREASE THE NUMBER OF RANDOM STARTS USING THE LRTSTARTS OPTION.
683
684
685
686       Beginning Time:  13:41:15
687          Ending Time:  13:41:45
688         Elapsed Time:  00:00:30
```

```
689
690
691 Mplus VERSION 8.8 DEMO (Mac) has the following limitations:
692   Maximum number of dependent variables: 6
693   Maximum number of independent variables: 2
694   Maximum number of between variables: 2
695   Maximum number of continuous latent variables in time series analysis: 2
696
697
698 MUTHEN & MUTHEN
699 3463 Stoner Ave.
700 Los Angeles, CA  90066
701
702 Tel: (310) 391-9971
703 Fax: (310) 391-8971
704 Web: www.StatModel.com
705 Support: Support@StatModel.com
706
707 Copyright (c) 1998-2022 Muthen & Muthen
```

ex0207.out

You need a good theory to guide model building



ex0208.inp

```
TITLE:     Growth mixture model, msq sum scores from EPESE
DATA:      FILE = ex0201.dat;
VARIABLE:  NAMES = msqtot1 msqtot4 msqtot7 age black ;
           MISSING ARE ALL (-9999) ;
           CLASSES = c(2)  ;
DEFINE:    CENTER age black (GRANDMEAN) ;
OUTPUT:    TECH1  TECH11 TECH14 ;
ANALYSIS:  TYPE = mixture  ;
MODEL:     %overall%
           i s | msqtot1@0 msqtot4@.6 msqtot7@1.2 ;
           i s on age black ;
           c#1 on age black  ;
           %c#1%
           msqtot1-msqtot7* (1) ;
           [i* s*] ;
           i* s* ;
           i s on age black ;
           %c#2%
           msqtot1-msqtot7* (2) ;
           [i* s*] ;
           i* s* ;
           i s on age black ;
```

```
Mplus VERSION 8.8 DEMO (Mac)
MUTHEN & MUTHEN
07/11/2022   1:41 PM

INPUT INSTRUCTIONS

  TITLE:     Growth mixture model, msq sum scores from EPESE
  DATA:      FILE = ex0201.dat;
  VARIABLE:  NAMES = msqtot1 msqtot4 msqtot7 age black ;
             MISSING ARE ALL (-9999) ;
             CLASSES = c(2)  ;
  DEFINE:    CENTER age black (GRANDMEAN) ;
  OUTPUT:    TECH1  TECH11 TECH14 ;
  ANALYSIS:  TYPE = mixture  ;
  MODEL:     %overall%
             i s | msqtot1@0 msqtot4@.6 msqtot7@1.2 ;
             i s on age black ;
             c#1 on age black  ;
             %c#1%
             msqtot1-msqtot7* (1) ;
             [i* s*] ;
             i* s* ;
             i s on age black ;
             %c#2%
             msqtot1-msqtot7* (2) ;
             [i* s*] ;
             i* s* ;
             i s on age black ;


*** WARNING
```

```
  Data set contains cases with missing on all variables except
  x-variables.  These cases were not included in the analysis.
  Number of cases with missing on all variables except x-variables:  397
   1 WARNING(S) FOUND IN THE INPUT INSTRUCTIONS



Growth mixture model, msq sum scores from EPESE

SUMMARY OF ANALYSIS

Number of groups                                                 1
Number of observations                                       14059

Number of dependent variables                                    3
Number of independent variables                                  2
Number of continuous latent variables                            2
Number of categorical latent variables                           1

Observed dependent variables

  Continuous
   MSQTOT1     MSQTOT4     MSQTOT7

Observed independent variables
   AGE         BLACK

Continuous latent variables
   I           S

Categorical latent variables
   C
```

```
65
66 Variables with special functions
67
68   Centering (GRANDMEAN)
69    AGE          BLACK
70
71
72 Estimator                                                    MLR
73 Information matrix                                      OBSERVED
74 Optimization Specifications for the Quasi-Newton Algorithm for
75 Continuous Outcomes
76   Maximum number of iterations                              100
77   Convergence criterion                              0.100D-05
78 Optimization Specifications for the EM Algorithm
79   Maximum number of iterations                              500
80   Convergence criteria
81     Loglikelihood change                             0.100D-06
82     Relative loglikelihood change                    0.100D-06
83     Derivative                                       0.100D-05
84 Optimization Specifications for the M step of the EM Algorithm for
85 Categorical Latent variables
86   Number of M step iterations                                 1
87   M step convergence criterion                     0.100D-05
88   Basis for M step termination                     ITERATION
89 Optimization Specifications for the M step of the EM Algorithm for
90 Censored, Binary or Ordered Categorical (Ordinal), Unordered
91 Categorical (Nominal) and Count Outcomes
92   Number of M step iterations                                 1
93   M step convergence criterion                     0.100D-05
94   Basis for M step termination                     ITERATION
95   Maximum value for logit thresholds                         15
96   Minimum value for logit thresholds                        -15
```

```
  Minimum expected cell size for chi-square              0.100D-01
Maximum number of iterations for H1                           2000
Convergence criterion for H1                            0.100D-03
Optimization algorithm                                        EMA
Random Starts Specifications
  Number of initial stage random starts                         20
  Number of final stage optimizations                            4
  Number of initial stage iterations                            10
  Initial stage convergence criterion                   0.100D+01
  Random starts scale                                   0.500D+01
  Random seed for generating random starts                       0

Input data file(s)
  ex0201.dat
Input data format  FREE


SUMMARY OF DATA

     Number of missing data patterns             7
     Number of y missing data patterns           7
     Number of u missing data patterns           0


COVARIANCE COVERAGE OF DATA

Minimum covariance coverage value   0.100


     PROPORTION OF DATA PRESENT FOR Y


```

```
129             Covariance Coverage
130                MSQTOT1        MSQTOT4        MSQTOT7        AGE            BLACK
131                --------       --------       --------       --------       --------
132   MSQTOT1        0.974
133   MSQTOT4        0.752          0.775
134   MSQTOT7        0.565          0.561          0.581
135   AGE            0.974          0.775          0.581          1.000
136   BLACK          0.974          0.775          0.581          1.000          1.000
137
138
139
140 UNIVARIATE SAMPLE STATISTICS
141
142
143      UNIVARIATE HIGHER-ORDER MOMENT DESCRIPTIVE STATISTICS
144
145          Variable/          Mean/     Skewness/   Minimum/ % with
                  Percentiles
146         Sample Size      Variance    Kurtosis    Maximum  Min/Max      20%/60%    40%/
                80%    Median
147
148      MSQTOT1                4.768      -1.152       0.000    0.70%       4.000
             5.000       5.000
149           13698.000         1.621       1.156       6.000   34.84%       5.000
                  6.000
150      MSQTOT4                4.732      -1.233       0.000    1.17%       4.000
             5.000       5.000
151           10899.000         1.822       1.268       6.000   35.05%       5.000
                  6.000
152      MSQTOT7                4.569      -1.160       0.000    2.00%       4.000
             5.000       5.000
```

```
153              8169.000        2.033        1.060        6.000    30.31%        5.000
                      6.000
154       AGE                    0.000        0.583       -6.932    31.36%       -6.932
             -1.932      -1.932
155              14059.000       40.275       -0.736       13.068     8.17%        3.068
                      8.068
156       BLACK                  0.000        1.528       -0.196    80.35%       -0.196
             -0.196      -0.196
157              14059.000        0.158        0.335        0.804    19.65%       -0.196
                     -0.196
158
159 RANDOM STARTS RESULTS RANKED FROM THE BEST TO THE WORST LOGLIKELIHOOD VALUES
160
161 1 perturbed starting value run(s) did not converge in the initial stage
162 optimizations.
163
164 Final stage loglikelihood values at local maxima, seeds, and initial stage start
          numbers:
165
166             -48186.170  93468            3
167             -48186.170  533738           11
168             -48186.170  76974            16
169             -48186.170  637345           19
170
171
172
173 THE BEST LOGLIKELIHOOD VALUE HAS BEEN REPLICATED.  RERUN WITH AT LEAST TWICE THE
174 RANDOM STARTS TO CHECK THAT THE BEST LOGLIKELIHOOD IS STILL OBTAINED AND REPLICATED.
175
176
177 THE MODEL ESTIMATION TERMINATED NORMALLY
178
```

```
179
180
181 MODEL FIT INFORMATION
182
183 Number of Free Parameters                       22
184
185 Loglikelihood
186
187           H0 Value                       -48186.170
188           H0 Scaling Correction Factor       1.3533
189             for MLR
190
191 Information Criteria
192
193           Akaike (AIC)                    96416.340
194           Bayesian (BIC)                  96582.462
195           Sample-Size Adjusted BIC        96512.548
196             (n* = (n + 2) / 24)
197
198
199
200 FINAL CLASS COUNTS AND PROPORTIONS FOR THE LATENT CLASSES
201 BASED ON THE ESTIMATED MODEL
202
203     Latent
204    Classes
205
206        1        7415.01070          0.52742
207        2        6643.98930          0.47258
208
209
210 FINAL CLASS COUNTS AND PROPORTIONS FOR THE LATENT CLASSES
```

```
BASED ON ESTIMATED POSTERIOR PROBABILITIES

    Latent
   Classes

       1       7415.01039          0.52742
       2       6643.98961          0.47258


FINAL CLASS COUNTS AND PROPORTIONS FOR THE LATENT CLASSES
BASED ON THEIR MOST LIKELY LATENT CLASS MEMBERSHIP

Class Counts and Proportions

    Latent
   Classes

       1            8126          0.57799
       2            5933          0.42201


CLASSIFICATION QUALITY

     Entropy                         0.571


Average Latent Class Probabilities for Most Likely Latent Class Membership (Row)
by Latent Class (Column)

           1        2

    1   0.846    0.154
```

```
    2    0.092    0.908


Classification Probabilities for the Most Likely Latent Class Membership (Column)
by Latent Class (Row)

             1        2

    1    0.927    0.073
    2    0.189    0.811


Logits for the Classification Probabilities for the Most Likely Latent Class Membership
       (Column)
by Latent Class (Row)

               1        2

    1      2.538    0.000
    2     -1.458    0.000


MODEL RESULTS

                                                    Two-Tailed
                    Estimate       S.E.  Est./S.E.    P-Value

Latent Class 1

 I        |
    MSQTOT1            1.000      0.000    999.000    999.000
    MSQTOT4            1.000      0.000    999.000    999.000
```

```
274     MSQTOT7            1.000      0.000    999.000    999.000
275
276  S        |
277     MSQTOT1            0.000      0.000    999.000    999.000
278     MSQTOT4            0.600      0.000    999.000    999.000
279     MSQTOT7            1.200      0.000    999.000    999.000
280
281  I          ON
282     AGE                0.000      0.002     -0.149      0.882
283     BLACK             -0.225      0.048     -4.717      0.000
284
285  S          ON
286     AGE               -0.015      0.002     -6.261      0.000
287     BLACK              0.085      0.040      2.134      0.033
288
289  S        WITH
290     I                  0.010      0.017      0.588      0.557
291
292  Intercepts
293     MSQTOT1            0.000      0.000    999.000    999.000
294     MSQTOT4            0.000      0.000    999.000    999.000
295     MSQTOT7            0.000      0.000    999.000    999.000
296     I                  5.469      0.055     99.439      0.000
297     S                 -0.191      0.028     -6.839      0.000
298
299  Residual Variances
300     MSQTOT1            0.342      0.024     14.082      0.000
301     MSQTOT4            0.342      0.024     14.082      0.000
302     MSQTOT7            0.342      0.024     14.082      0.000
303     I                  0.051      0.027      1.847      0.065
304     S                  0.076      0.016      4.657      0.000
305
```

```
Latent Class 2

 I        |
    MSQTOT1             1.000      0.000    999.000    999.000
    MSQTOT4             1.000      0.000    999.000    999.000
    MSQTOT7             1.000      0.000    999.000    999.000

 S        |
    MSQTOT1             0.000      0.000    999.000    999.000
    MSQTOT4             0.600      0.000    999.000    999.000
    MSQTOT7             1.200      0.000    999.000    999.000

 I          ON
    AGE                -0.039      0.003    -13.027      0.000
    BLACK              -0.539      0.046    -11.596      0.000

 S          ON
    AGE                -0.037      0.005     -8.147      0.000
    BLACK              -0.153      0.057     -2.706      0.007

 S        WITH
    I                   0.010      0.017      0.588      0.557

 Intercepts
    MSQTOT1             0.000      0.000    999.000    999.000
    MSQTOT4             0.000      0.000    999.000    999.000
    MSQTOT7             0.000      0.000    999.000    999.000
    I                   4.123      0.041    100.657      0.000
    S                  -0.483      0.053     -9.055      0.000

 Residual Variances
    MSQTOT1             1.311      0.055     23.916      0.000
```

```
    MSQTOT4            1.311      0.055     23.916      0.000
    MSQTOT7            1.311      0.055     23.916      0.000
    I                  0.486      0.038     12.684      0.000
    S                  0.336      0.042      7.959      0.000

Categorical Latent Variables

 C#1        ON
    AGE               -0.080      0.007    -10.995      0.000
    BLACK             -0.954      0.079    -12.120      0.000

 Intercepts
    C#1                0.110      0.147      0.746      0.456


QUALITY OF NUMERICAL RESULTS

     Condition Number for the Information Matrix              0.434E-03
       (ratio of smallest to largest eigenvalue)


LOGISTIC REGRESSION ODDS RATIO RESULTS

                                                 95% C.I.
                   Estimate       S.E.  Lower 2.5% Upper 2.5%

Categorical Latent Variables

 C#1      ON
    AGE                0.923      0.007      0.910      0.937
    BLACK              0.385      0.030      0.330      0.450

```

```
370
371 ALTERNATIVE PARAMETERIZATIONS FOR THE CATEGORICAL LATENT VARIABLE REGRESSION
372
373                                                          Two-Tailed
374                     Estimate       S.E.  Est./S.E.    P-Value
375
376 Parameterization using Reference Class 1
377
378  C#2      ON
379     AGE                 0.080      0.007     10.995      0.000
380     BLACK               0.954      0.079     12.120      0.000
381
382  Intercepts
383     C#2                -0.110      0.147     -0.746      0.456
384
385
386 ODDS RATIO FOR THE ALTERNATIVE PARAMETERIZATIONS FOR THE CATEGORICAL LATENT VARIABLE
       REGRESSION
387
388                                                 95% C.I.
389                     Estimate       S.E.  Lower 2.5% Upper 2.5%
390
391 Parameterization using Reference Class 1
392
393  C#2      ON
394     AGE                 1.083      0.008      1.068      1.098
395     BLACK               2.595      0.204      2.224      3.028
396
397
398 TECHNICAL 1 OUTPUT
399
400
```

```
401       PARAMETER SPECIFICATION FOR LATENT CLASS 1
402
403
404             NU
405                MSQTOT1       MSQTOT4       MSQTOT7       AGE           BLACK
406                --------      --------      --------      --------      --------
407                       0             0             0             0             0
408
409
410             LAMBDA
411                I             S             AGE           BLACK
412                --------      --------      --------      --------
413   MSQTOT1             0             0             0             0
414   MSQTOT4             0             0             0             0
415   MSQTOT7             0             0             0             0
416   AGE                 0             0             0             0
417   BLACK               0             0             0             0
418
419
420             THETA
421                MSQTOT1       MSQTOT4       MSQTOT7       AGE           BLACK
422                --------      --------      --------      --------      --------
423   MSQTOT1             1
424   MSQTOT4             0             1
425   MSQTOT7             0             0             1
426   AGE                 0             0             0             0
427   BLACK               0             0             0             0             0
428
429
430             ALPHA
431                I             S             AGE           BLACK
432                --------      --------      --------      --------
```

```
                    2              3              0              0

           BETA
              I              S              AGE            BLACK
              ________       ________       ________       ________
I                   0              0              4              5
S                   0              0              6              7
AGE                 0              0              0              0
BLACK               0              0              0              0

           PSI
              I              S              AGE            BLACK
              ________       ________       ________       ________
I                   8
S                   9             10
AGE                 0              0              0
BLACK               0              0              0              0

    PARAMETER SPECIFICATION FOR LATENT CLASS 2

           NU
              MSQTOT1        MSQTOT4        MSQTOT7        AGE            BLACK
              ________       ________       ________       ________       ________
                    0              0              0              0              0

           LAMBDA
              I              S              AGE            BLACK
```

```
                  --------      --------      --------      --------
MSQTOT1                0             0             0             0
MSQTOT4                0             0             0             0
MSQTOT7                0             0             0             0
AGE                    0             0             0             0
BLACK                  0             0             0             0


           THETA
              MSQTOT1       MSQTOT4       MSQTOT7       AGE           BLACK
              --------      --------      --------      --------      --------
MSQTOT1            11
MSQTOT4             0            11
MSQTOT7             0             0            11
AGE                 0             0             0             0
BLACK               0             0             0             0             0


           ALPHA
              I             S             AGE           BLACK
              --------      --------      --------      --------
                   12            13             0             0


           BETA
              I             S             AGE           BLACK
              --------      --------      --------      --------
I                   0             0            14            15
S                   0             0            16            17
AGE                 0             0             0             0
BLACK               0             0             0             0
```

```
497
498             PSI
499                I              S              AGE            BLACK
500                --------       --------       --------       --------
501  I                  18
502  S                   9             19
503  AGE                 0              0              0
504  BLACK               0              0              0              0
505
506
507       PARAMETER SPECIFICATION FOR LATENT CLASS REGRESSION MODEL PART
508
509
510             ALPHA(C)
511                C#1            C#2
512                --------       --------
513                     20             0
514
515
516             GAMMA(C)
517                AGE            BLACK
518                --------       --------
519  C#1               21             22
520  C#2                0              0
521
522
523       STARTING VALUES FOR LATENT CLASS 1
524
525
526             NU
527                MSQTOT1        MSQTOT4        MSQTOT7        AGE            BLACK
528                --------       --------       --------       --------       --------
```

```
                0.000         0.000         0.000         0.000         0.000


           LAMBDA
              I             S             AGE           BLACK
              ________      ________      ________      ________
MSQTOT1         1.000         0.000         0.000         0.000
MSQTOT4         1.000         0.600         0.000         0.000
MSQTOT7         1.000         1.200         0.000         0.000
AGE             0.000         0.000         1.000         0.000
BLACK           0.000         0.000         0.000         1.000


           THETA
              MSQTOT1       MSQTOT4       MSQTOT7       AGE           BLACK
              ________      ________      ________      ________      ________
MSQTOT1         0.810
MSQTOT4         0.000         0.911
MSQTOT7         0.000         0.000         1.016
AGE             0.000         0.000         0.000         0.000
BLACK           0.000         0.000         0.000         0.000         0.000


           ALPHA
              I             S             AGE           BLACK
              ________      ________      ________      ________
                4.948        -0.406         0.000         0.000


           BETA
              I             S             AGE           BLACK
              ________      ________      ________      ________
```

```
I                0.000          0.000          0.000          0.000
S                0.000          0.000          0.000          0.000
AGE              0.000          0.000          0.000          0.000
BLACK            0.000          0.000          0.000          0.000


           PSI
              I              S              AGE            BLACK
              ________       ________       ________       ________
I               1.247
S               0.000          2.157
AGE             0.000          0.000         20.138
BLACK           0.000          0.000          0.000          0.079


    STARTING VALUES FOR LATENT CLASS 2


           NU
              MSQTOT1        MSQTOT4        MSQTOT7        AGE            BLACK
              ________       ________       ________       ________       ________
                0.000          0.000          0.000          0.000          0.000


           LAMBDA
              I              S              AGE            BLACK
              ________       ________       ________       ________
MSQTOT1         1.000          0.000          0.000          0.000
MSQTOT4         1.000          0.600          0.000          0.000
MSQTOT7         1.000          1.200          0.000          0.000
AGE             0.000          0.000          1.000          0.000
BLACK           0.000          0.000          0.000          1.000
```

```
593
594
595             THETA
596                MSQTOT1       MSQTOT4       MSQTOT7       AGE           BLACK
597                --------      --------      --------      --------      --------
598   MSQTOT1        0.810
599   MSQTOT4        0.000         0.911
600   MSQTOT7        0.000         0.000         1.016
601   AGE            0.000         0.000         0.000         0.000
602   BLACK          0.000         0.000         0.000         0.000         0.000
603
604
605             ALPHA
606                I             S             AGE           BLACK
607                --------      --------      --------      --------
608                4.948        -0.406         0.000         0.000
609
610
611             BETA
612                I             S             AGE           BLACK
613                --------      --------      --------      --------
614   I              0.000         0.000         0.000         0.000
615   S              0.000         0.000         0.000         0.000
616   AGE            0.000         0.000         0.000         0.000
617   BLACK          0.000         0.000         0.000         0.000
618
619
620             PSI
621                I             S             AGE           BLACK
622                --------      --------      --------      --------
623   I              1.247
624   S              0.000         2.157
```

```
AGE              0.000          0.000         20.138
BLACK            0.000          0.000          0.000          0.079


     STARTING VALUES FOR LATENT CLASS REGRESSION MODEL PART


           ALPHA(C)
              C#1            C#2
              --------       --------
                0.000          0.000


           GAMMA(C)
              AGE            BLACK
              --------       --------
C#1             0.000          0.000
C#2             0.000          0.000


TECHNICAL 11 OUTPUT

     Random Starts Specifications for the k-1 Class Analysis Model
        Number of initial stage random starts                    20
        Number of final stage optimizations                       4



     VUONG-LO-MENDELL-RUBIN LIKELIHOOD RATIO TEST FOR 1 (H0) VERSUS 2 CLASSES

          H0 Loglikelihood Value                       -51460.800
          2 Times the Loglikelihood Difference           6549.260
```

```
          Difference in the Number of Parameters                12
          Mean                                                80.097
          Standard Deviation                                 106.180
          P-Value                                             0.0000

     LO-MENDELL-RUBIN ADJUSTED LRT TEST

          Value                                            6492.612
          P-Value                                             0.0000


TECHNICAL 14 OUTPUT

     Random Starts Specifications for the k-1 Class Analysis Model
        Number of initial stage random starts                   20
        Number of final stage optimizations                      4

     Random Starts Specification for the k-1 Class Model for Generated Data
        Number of initial stage random starts                    0
        Number of final stage optimizations for the
           initial stage random starts                           0
     Random Starts Specification for the k Class Model for Generated Data
        Number of initial stage random starts                   40
        Number of final stage optimizations                      8
     Number of bootstrap draws requested                     Varies


     PARAMETRIC BOOTSTRAPPED LIKELIHOOD RATIO TEST FOR 1 (H0) VERSUS 2 CLASSES

          H0 Loglikelihood Value                          -51460.800
          2 Times the Loglikelihood Difference              6549.260
          Difference in the Number of Parameters                12
```

```
          Approximate P-Value                                  0.0000
          Successful Bootstrap Draws                                5

     WARNING:  OF THE 5 BOOTSTRAP DRAWS, 6 DRAWS HAD BOTH A SMALLER LRT VALUE THAN THE
     OBSERVED LRT VALUE AND NOT A REPLICATED BEST LOGLIKELIHOOD VALUE FOR THE 2-CLASS
          MODEL.
     THIS MEANS THAT THE P-VALUE MAY NOT BE TRUSTWORTHY DUE TO LOCAL MAXIMA.
     INCREASE THE NUMBER OF RANDOM STARTS USING THE LRTSTARTS OPTION.


     WARNING:  1 OUT OF 6 BOOTSTRAP DRAWS DID NOT CONVERGE.
     INCREASE THE NUMBER OF RANDOM STARTS USING THE LRTSTARTS OPTION.



     Beginning Time:  13:41:46
        Ending Time:  13:43:06
       Elapsed Time:  00:01:20


Mplus VERSION 8.8 DEMO (Mac) has the following limitations:
  Maximum number of dependent variables: 6
  Maximum number of independent variables: 2
  Maximum number of between variables: 2
  Maximum number of continuous latent variables in time series analysis: 2


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```

Interpretation of Results

- Two classes (53/47)
- Low entropy
  - too few repeated observations for reliable classification
- Class 1: Start high (5.5) go slow (-0.2/5y)
  - B/AA go slower

$$
\begin{aligned}
E(\alpha_s|\overline{age}, \text{black=1}, c=1) &= -0.191 + (\text{black} - \overline{\text{black}})0.085 \\
&= -0.191 + (1 - .193)0.085 \\
&= -0.122 \\
E(\alpha_s|\overline{age}, \text{black=0}, c=1) &= -0.191 + (\text{black} - \overline{\text{black}})0.085 \\
&= -0.191 + (0 - .193)0.085 \\
&= -0.207
\end{aligned}
$$

Interpretation of Results (con't)

- Class 2: Start lower(4.05) go faster (-0.5/5y)
  - B/AA go faster

$$
\begin{aligned}
E(\alpha_s|\overline{age}, \text{black=1}, c=2) &= -0.483 + (\text{black} - \overline{\text{black}})(-0.153) \\
&= -0.483 + (1 - .193)(-0.153) \\
&= -0.606 \\
E(\alpha_s|\overline{age}, \text{black=0}, c=1) &= -0.483 + (\text{black} - \overline{\text{black}})(-0.153) \\
&= -0.483 + (0 - .193)(-0.153) \\
&= -0.453
\end{aligned}
$$

Interpretation of Results (con't)

- Odds of being in class 2 vs. class 1
  - higher for B/AA
    - ⋆ Mplus gives us the odds ratio (OR) for class 1 vs. class 2 for unit increase in x
    - ⋆ for B/AA is OR $= 0.385$
    - ⋆ OR for class 2 vs. class 1 for B/AA is the inverse
    - ⋆ $OR = 0.385^{-1} = 2.5$
- It's complicated.

Lab Session
Mixture Models
(Mplus only)

Growth Modeling
Special Considerations

Growth Modeling
Multilevel Approach
Multiple Indicator Models
Other Issues
(retest effects, methods artifacts)
(Mplus only)

Day 3:

Multiple group models, randomized trials,
complex sampling weights, accelerated longitudinal designs,
latent change score model (bivariate change score),

survival analysis.

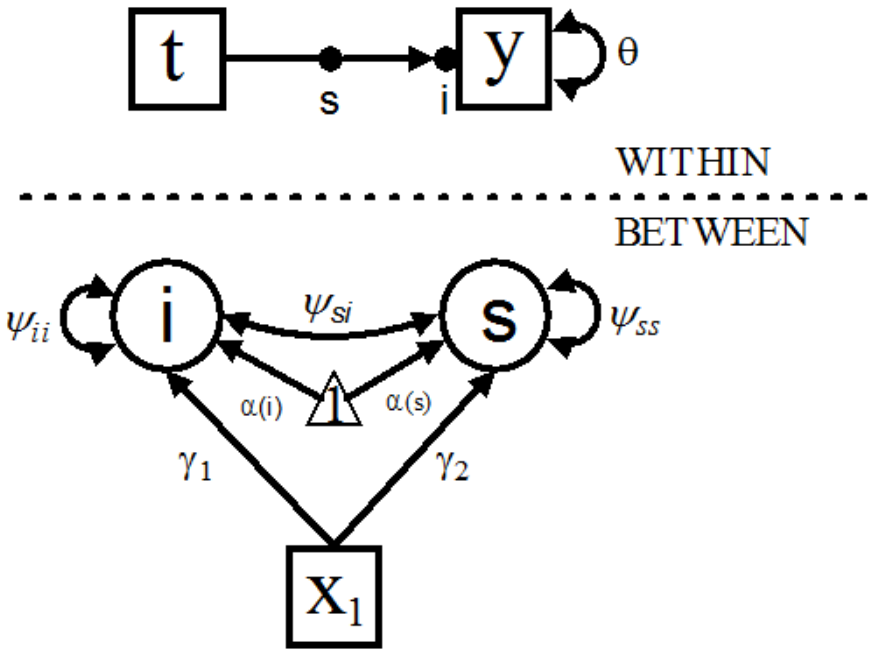
Multilevel Approach in Mplus

- We have covered latent variable approach
- An alternative approach is to use multilevel modeling capabilities
  - Long data layout
  - Two levels

    WITHIN repeated observations within a person, and time varying covariates

    BETWEEN factors capturing systematic change across persons over time

Multilevel Model

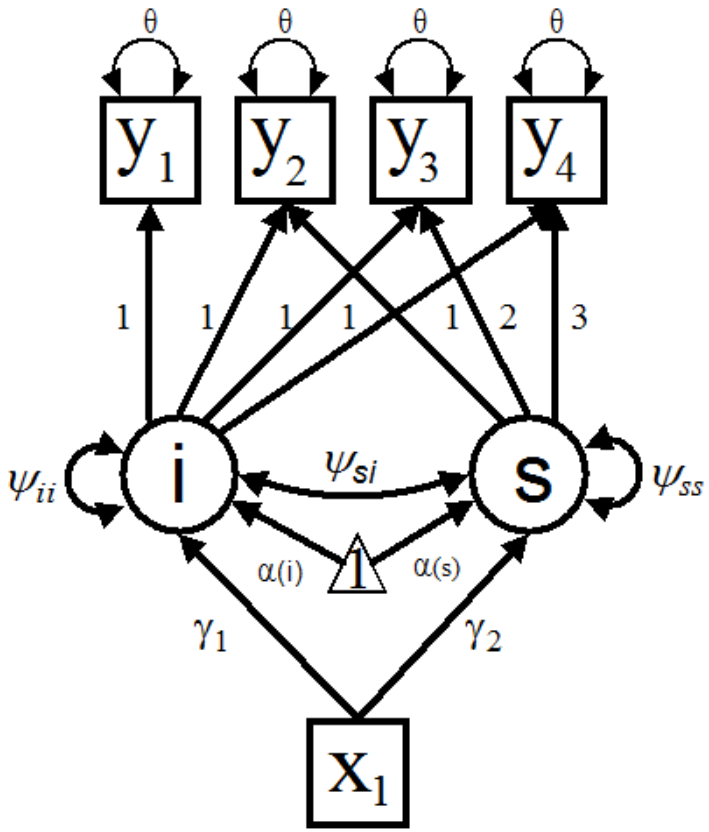


$$y_{it} = \mathbf{i}_i + \mathbf{s}_i t_i + \epsilon_i$$

$$\mathbf{i}_i = \alpha(\mathbf{i}) + x_{1i}\gamma_1 + \zeta(\mathbf{i})_i$$

$$\mathbf{s}_i = \alpha(\mathbf{s}) + x_{1i}\gamma_2 + \zeta(\mathbf{s})_i$$

LGMC



$$y_{it} = \mathbf{i}_i + \mathbf{s}_i t + \epsilon_i$$

$$\mathbf{i}_i = \alpha(\mathbf{i}) + x_{1i}\gamma_1 + \zeta(\mathbf{i})_i$$

$$\mathbf{s}_i = \alpha(\mathbf{s}) + x_{1i}\gamma_2 + \zeta(\mathbf{s})_i$$

```
TITLE:    Latent Growth Curve Model (ex02-03a.inp)
DATA:     FILE = ex02-03a.dat ;
VARIABLE: NAMES=msqtot1 msqtot4 msqtot7 age1 age4 age7 male black;
          MISSING ARE ALL (-9999) ;
          TSCORES = age1 age4 age7 ;
ANALYSIS: TYPE=random  ;
MODEL:    i s | msqtot1 msqtot4 msqtot7 at age1 age4 age7  ;
          i s on male black  ;
          msqtot1-msqtot7 (1) ;
=============================================================
TITLE:    Multilevel Growth Model (ex02-03b.inp)
DATA:     FILE = ex02-03b.dat ;
VARIABLE: NAMES = id age msqtot male black ;
          MISSING ARE ALL (-9999) ;
          WITHIN = age  ;
          BETWEEN = male black  ;
          CLUSTER = id ;
ANALYSIS: TYPE = twolevel random ;
MODEL:    %within%
             s | msqtot on age  ;
          %between%
             msqtot on male black  ;
             s on male black  ;
             msqtot with s ;
```

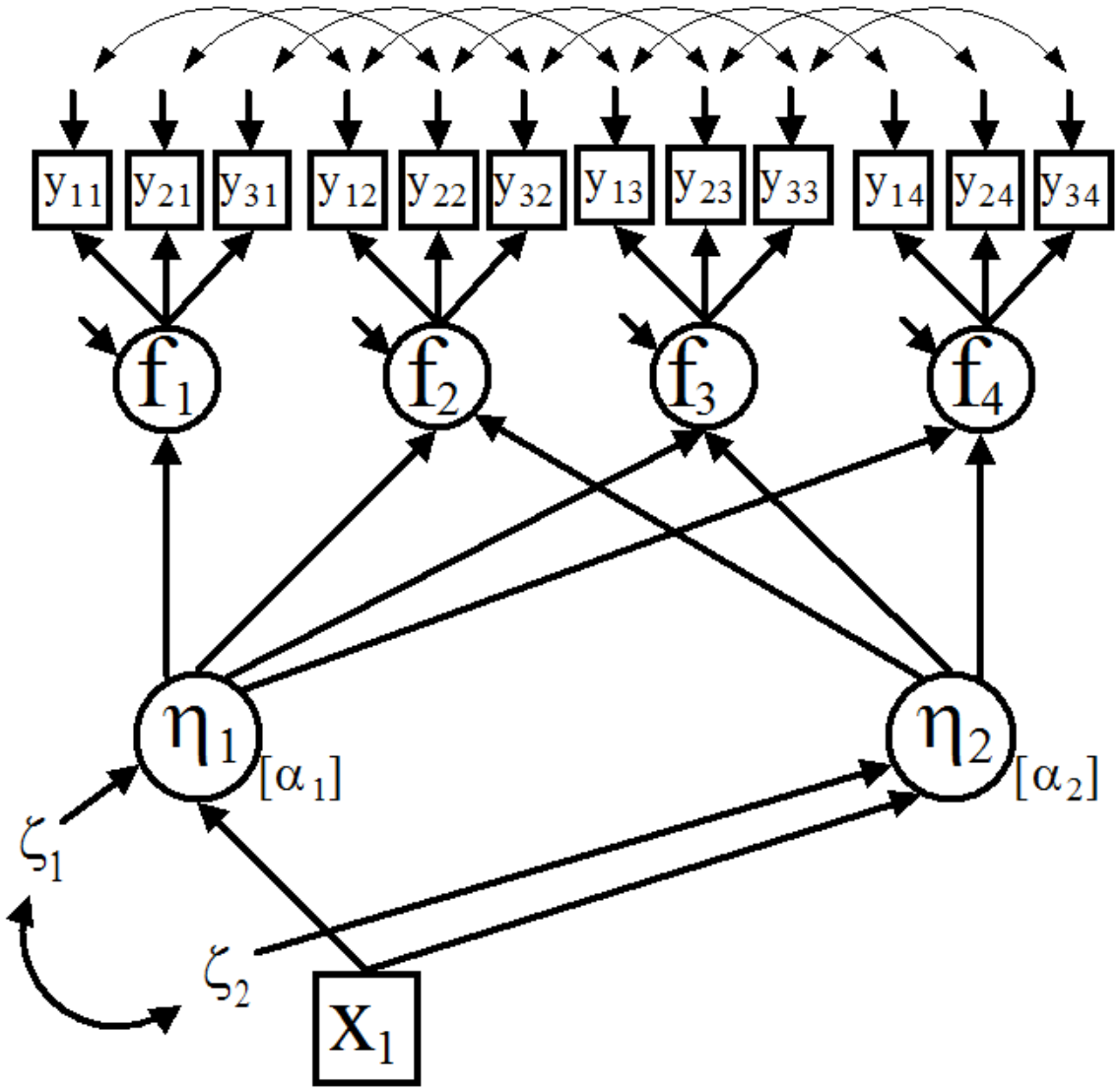
Mplus Multilevel Model

	Estimate	S.E.	Est./S.E.
Within Level			
Residual Variances			
MSQTOT	0.814	0.012	68.280
Between Level			
S ON			
MALE	0.050	0.014	3.490
BLACK	-0.060	0.019	-3.109
MSQTOT ON			
MALE	0.083	0.018	4.592
BLACK	-0.727	0.026	-27.664
MSQTOT WITH			
S	0.114	0.008	14.287
Intercepts			
MSQTOT	4.886	0.012	406.846
S	-0.275	0.009	-31.243
Residual Variances			
MSQTOT	0.613	0.021	29.479
S	0.084	0.010	8.788

Mplus LGCM

	Estimate	S.E.	Est./S.E.
Residual Variances			
MSQTOT1	0.814	0.012	68.279
MSQTOT4	0.814	0.012	68.279
MSQTOT7	0.814	0.012	68.279
S ON			
MALE	0.050	0.014	3.487
BLACK	-0.060	0.019	-3.115
I ON			
MALE	0.083	0.018	4.593
BLACK	-0.727	0.026	-27.664
S WITH			
I	0.115	0.008	14.296
Intercepts			
I	4.886	0.012	406.839
S	-0.275	0.009	-31.236
Residual Variances			
I	0.613	0.021	29.478
S	0.084	0.010	8.793
Intercepts			
MSQTOT1-7	0.000	0.000	999.000

Multiple Indicator Growth Model



Re-Test Effects (and other methods artifacts)

See

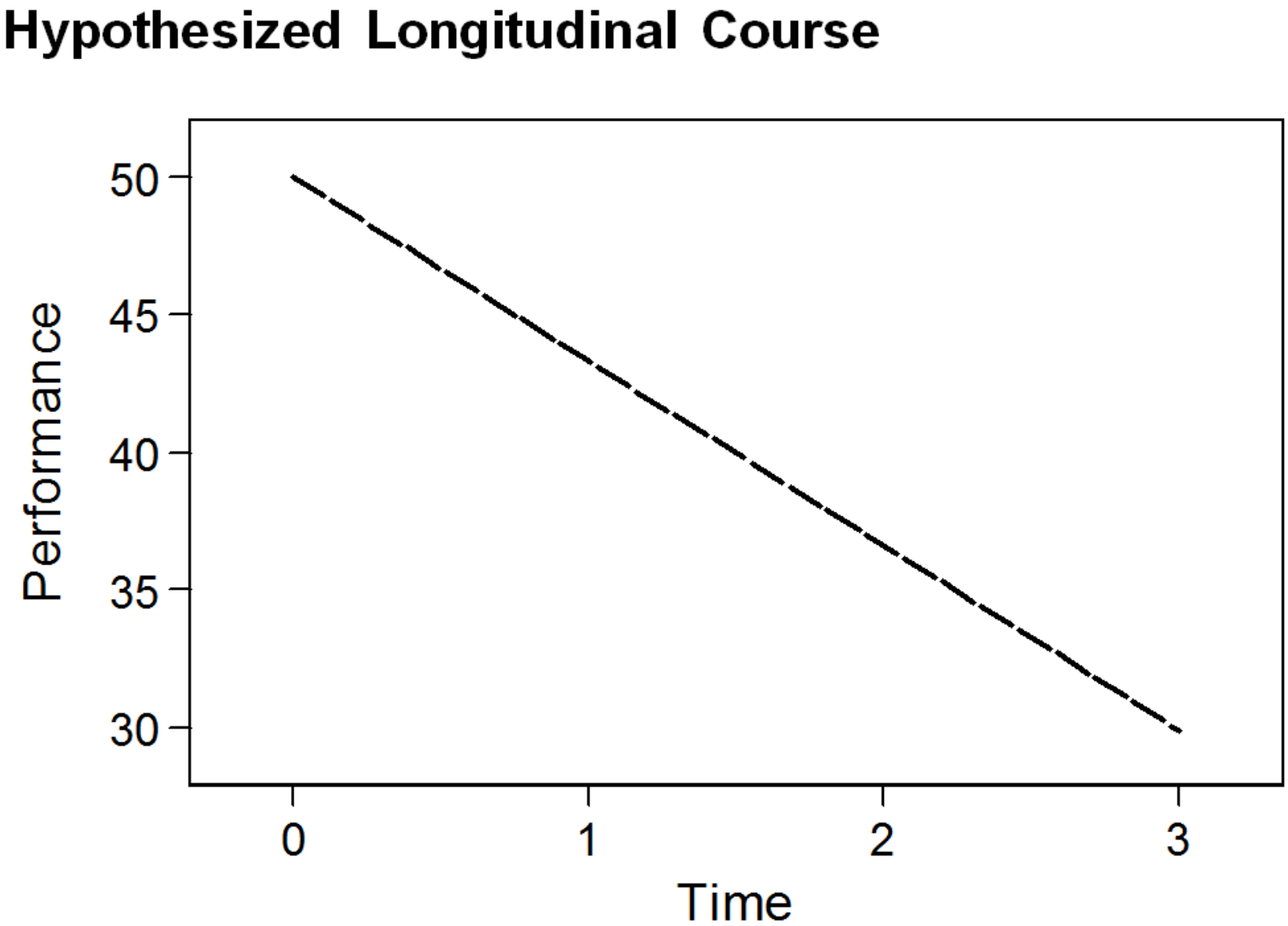
Ferrer E, Salthouse T, McArdle J, Stewart W, Schwartz B. Multivariate modeling of age and retest in longitudinal studies of cognitive abilities. *Psychology & Aging.* 2005;20(3):412.

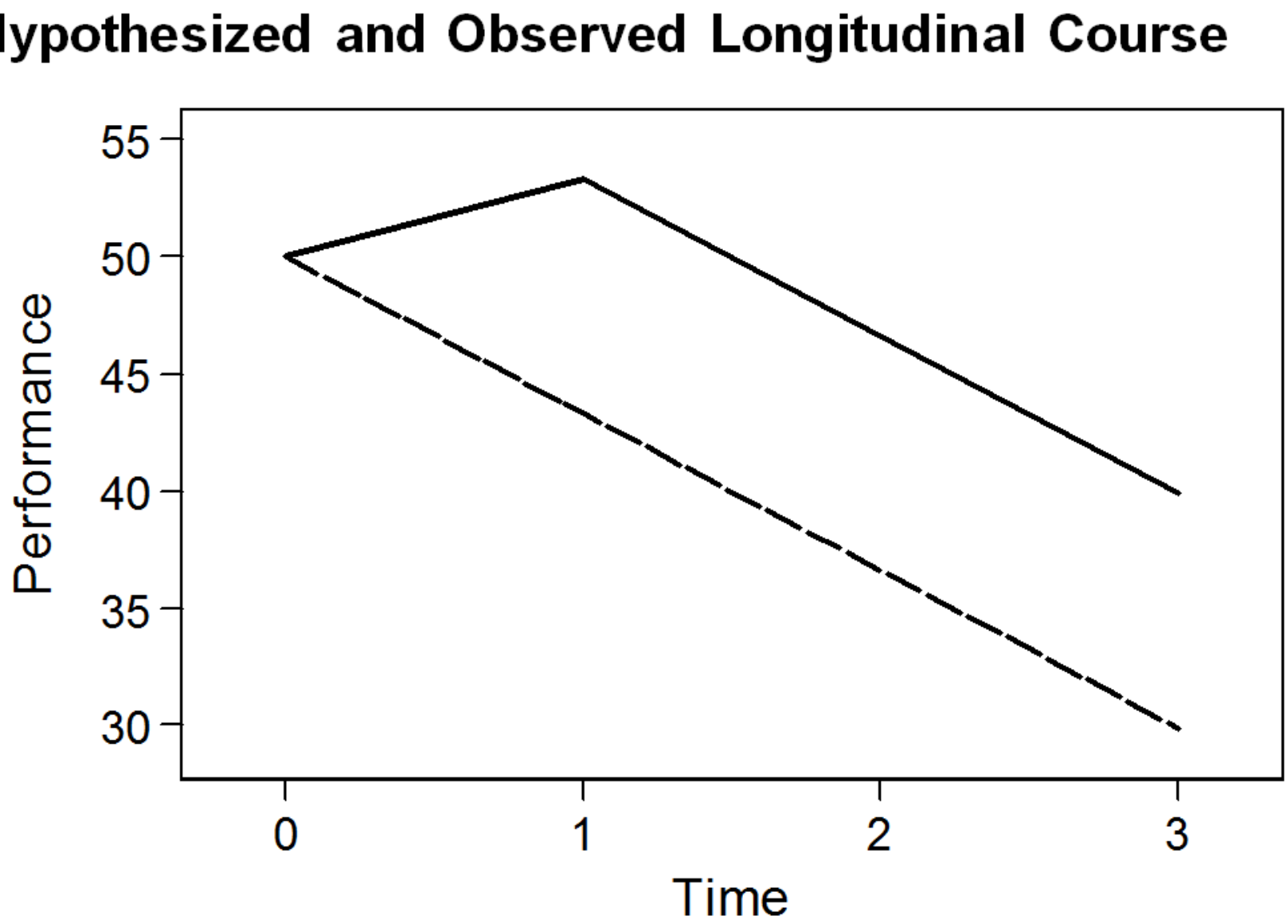
Ferrer E, Salthouse TA, Stewart WF, Schwartz BS. Modeling age and retest processes in longitudinal studies of cognitive abilities. *Psychology & Aging.* Jun 2004;19(2):243-259.

Salthouse T, Schroeder D, Ferrer E. Estimating retest effects in longitudinal assessments of cognitive functioning in adults between 18 and 60 years of age. *Developmental Psychology.* 2004;40(5):813-8229.

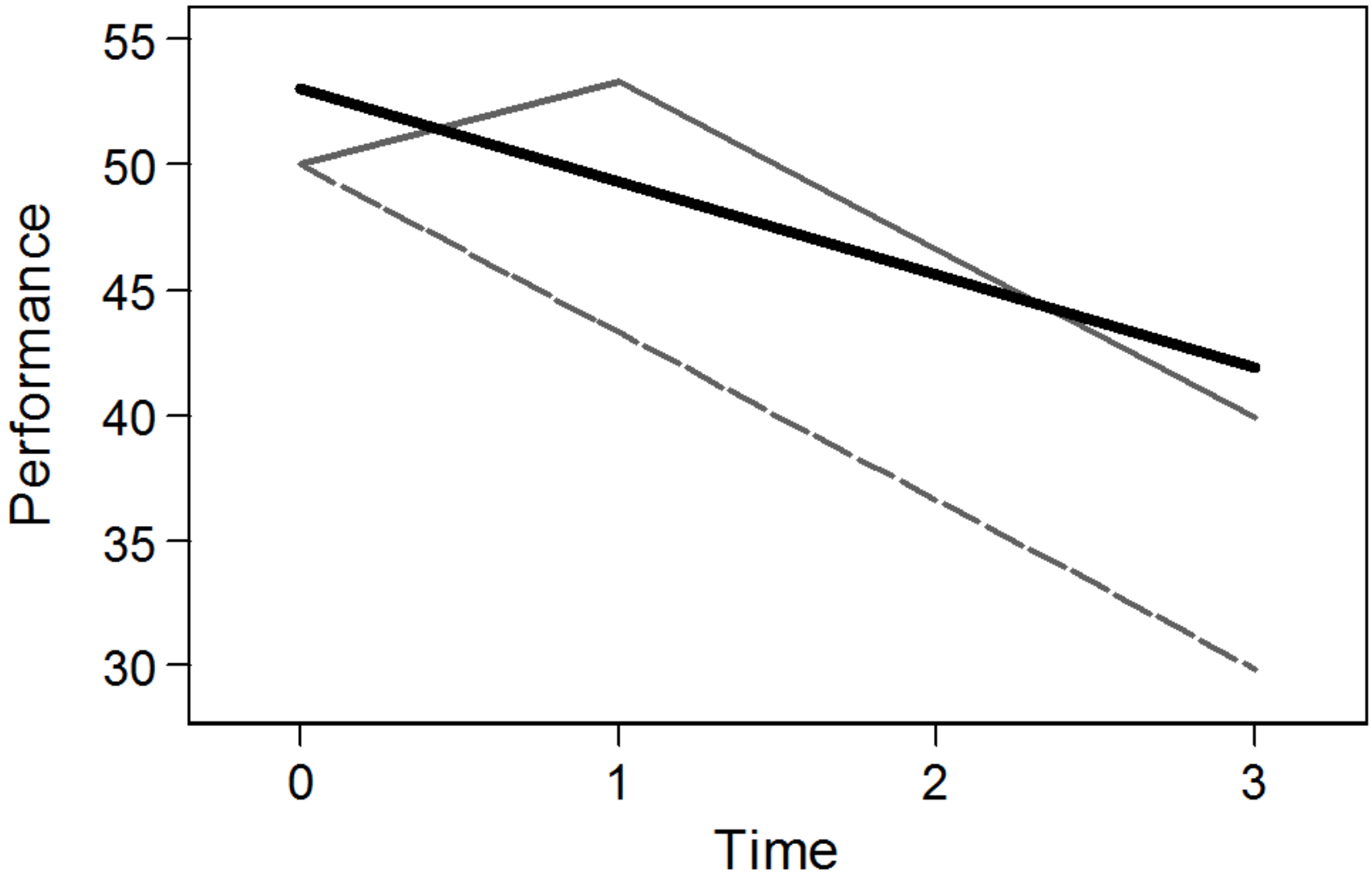
Rabbitt P, Diggle P, Holland F, McInnes L. Practice and drop-out effects during a 17-year longitudinal study of cognitive aging. *J Gerontol B Psychol Sci Soc Sci.* Mar 2004;59(2):P84-97.

Wilson RS, Li Y, Bienias JL, Bennett DA. Cognitive decline in old age: separating retest effects from the effects of growing older. *Psychology and aging.* 2006;21(4):774.

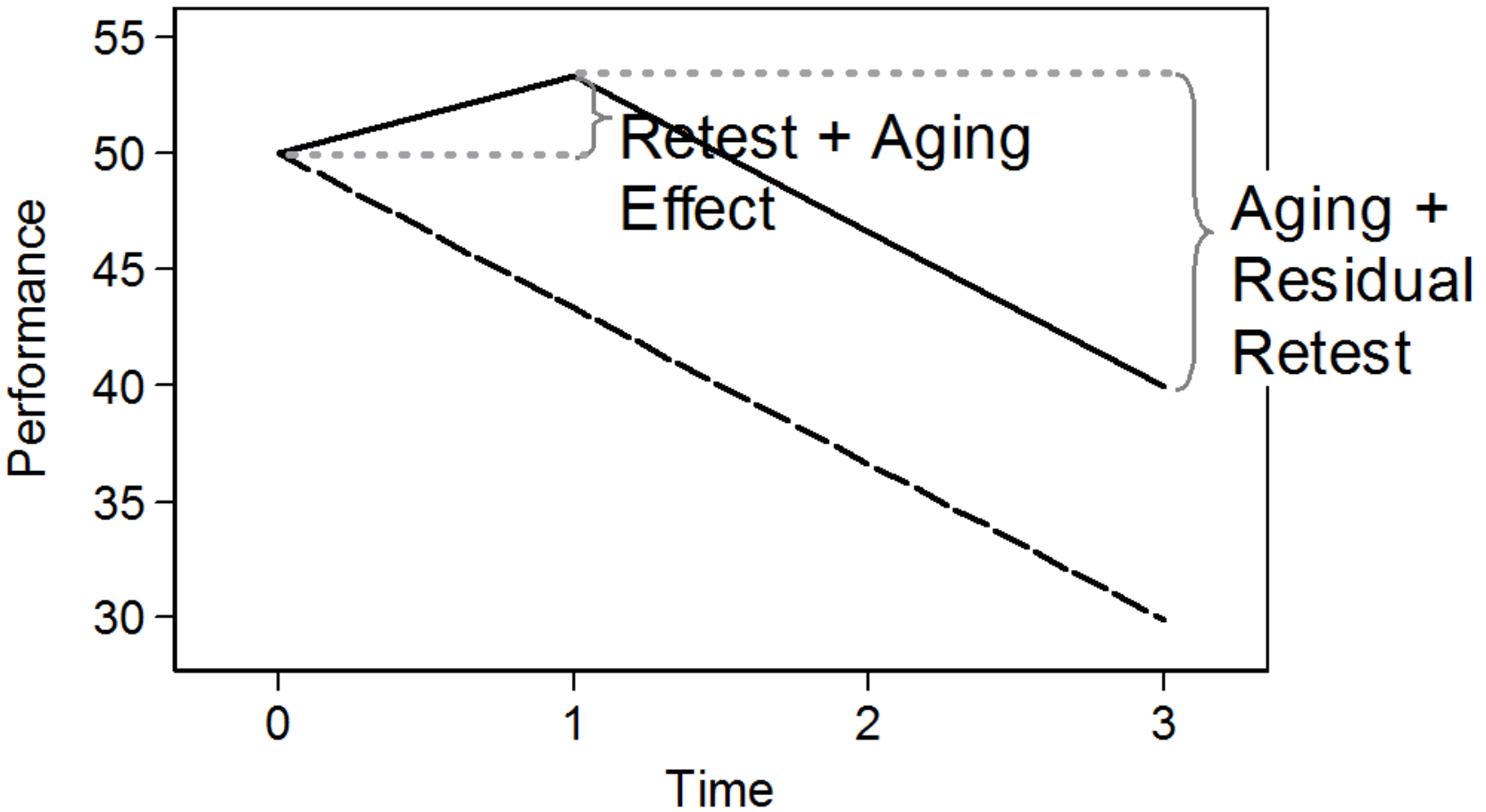

Hypothesized Longitudinal Course
Performance
50
45
40
35
30
0
1
2
3
Time


Hypothesized and Observed Longitudinal Course
Performance
55
50
45
40
35
30
0
1
2
3
Time

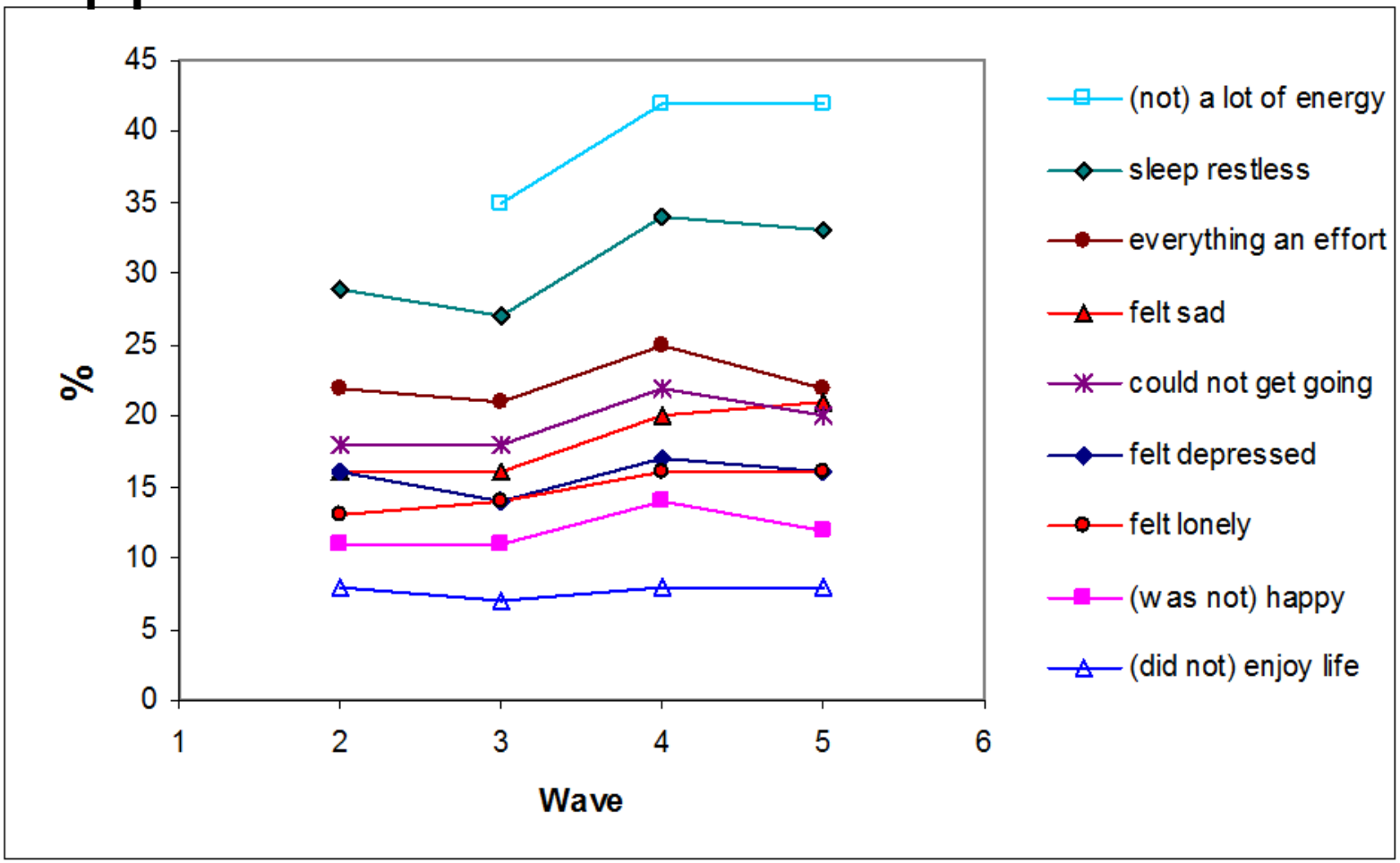
Bias in Estimate of Baseline Level and Change



Hypothesized Longitudinal Course

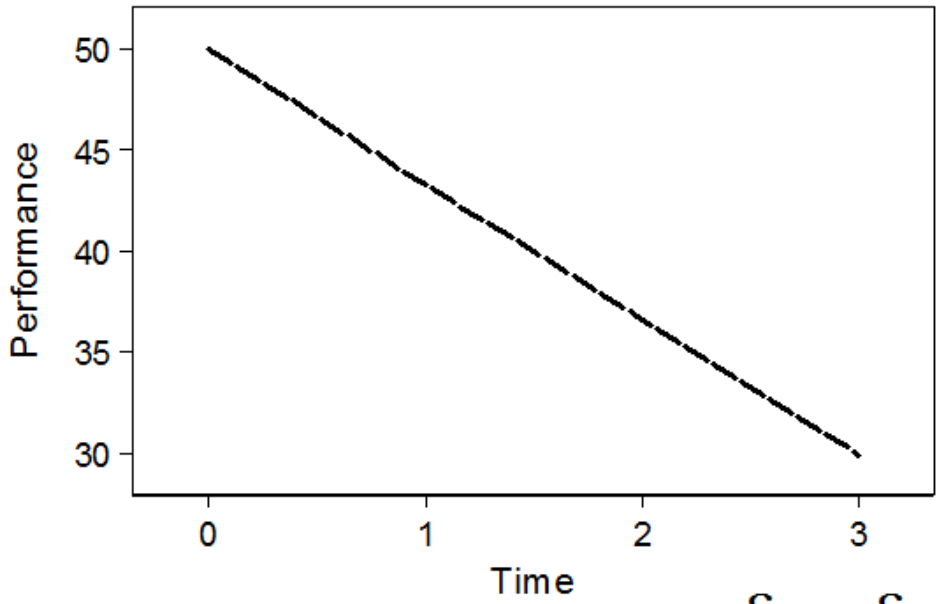


Applicable to Other Methods Artifacts



Profile of Depression Symptom Endorsement in HRS. Depression section moved between Wave 3 and 4, from health section to cognition section of interview, where used as a distractor during memory span testing (possible negative priming).

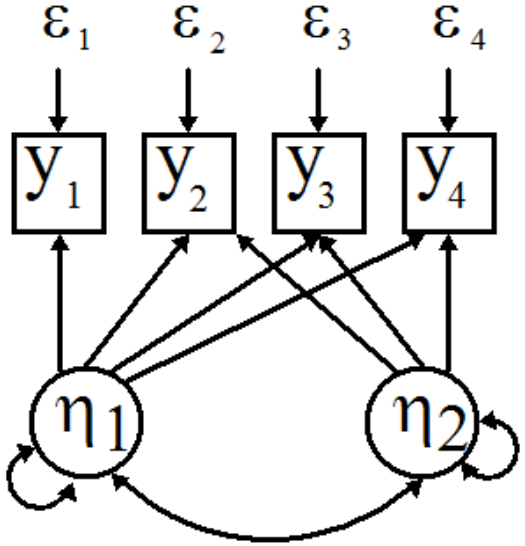
Latent Growth Model



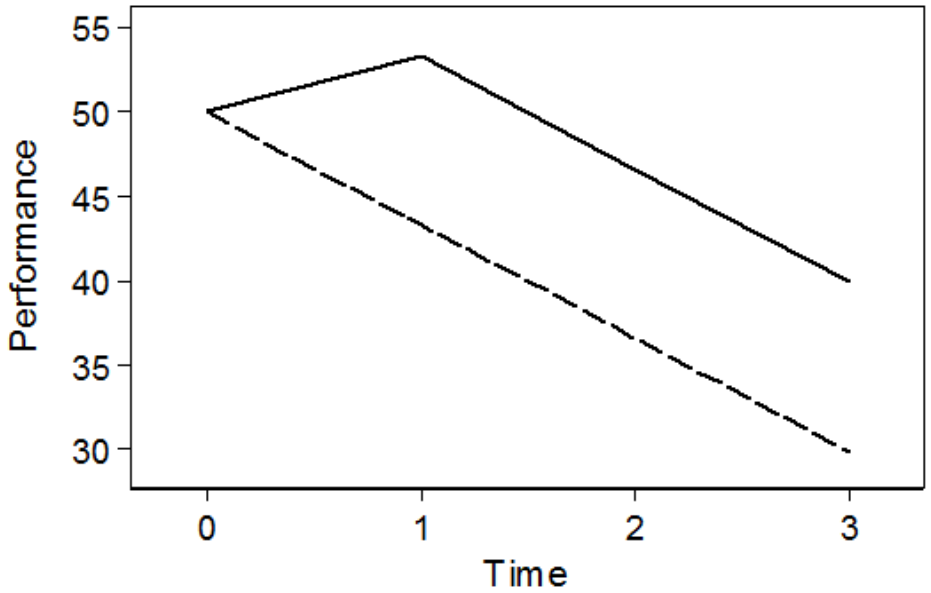
$$y_{ij} = \eta_{1i} + \eta_{2i} \times TIME_i + \varepsilon_{ij}$$

$$\eta_{1i} = \alpha_1 + \zeta_{1i}$$

$$\eta_{2i} = \alpha_2 + \zeta_{2i}$$



Hypothesized Longitudinal Course

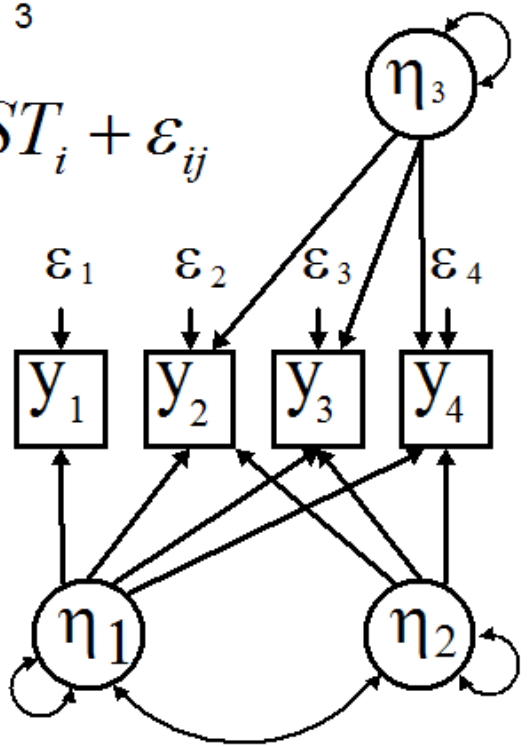


$$y_{ij} = \eta_{1i} + \eta_{2i} \times TIME_i + \eta_{3i} \times RETEST_i + \varepsilon_{ij}$$

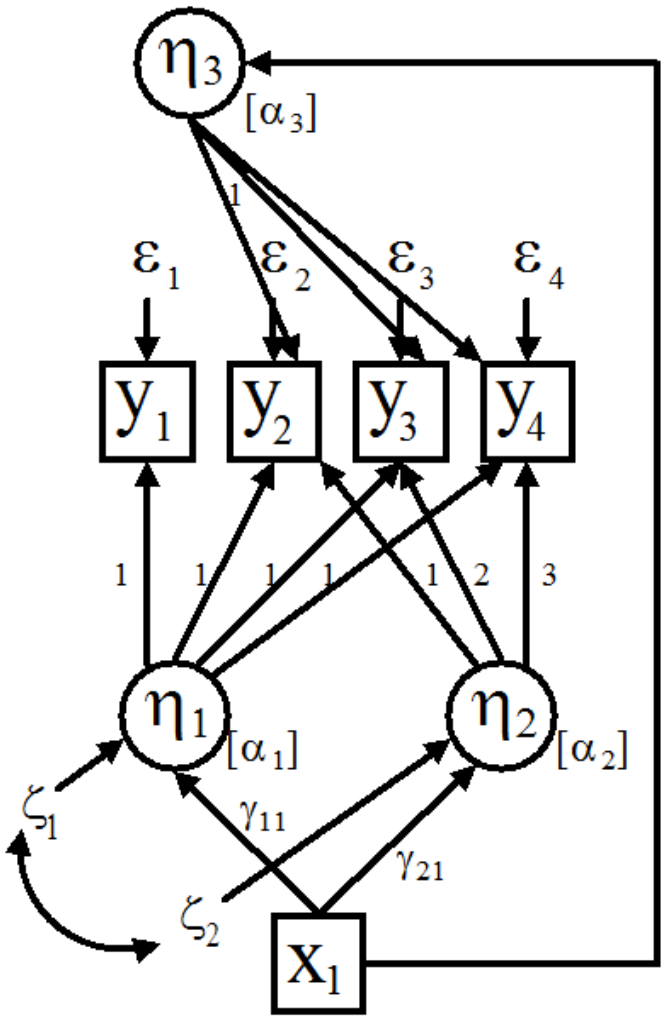
$$\eta_{1i} = \alpha_1 + \zeta_{1i}$$

$$\eta_{2i} = \alpha_2 + \zeta_{2i}$$

$$\eta_{3i} = \alpha_3 + \zeta_{3i}$$



Model a Retest Effect



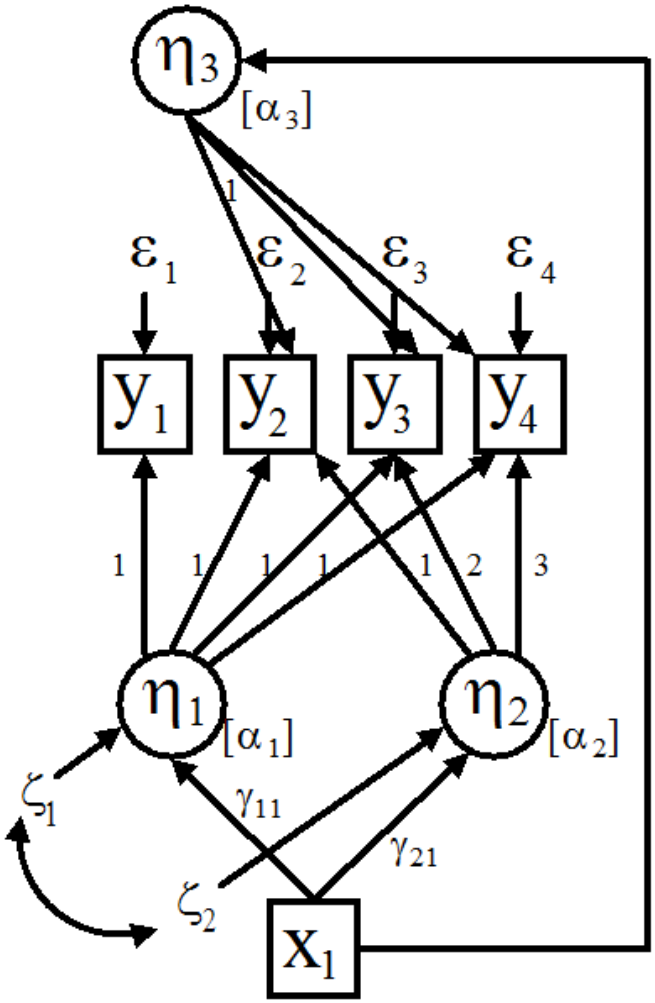
$$\nu = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \qquad \Lambda = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 3 & 1 \end{bmatrix}$$

$$\theta = VAR(\varepsilon) = \begin{bmatrix} * & 0 & 0 & 0 \\ 0 & * & 0 & 0 \\ 0 & 0 & * & 0 \\ 0 & 0 & 0 & * \end{bmatrix}$$

$$\alpha = \begin{bmatrix} * \\ * \\ * \end{bmatrix} \qquad \Gamma = \begin{bmatrix} * \\ * \\ * \end{bmatrix}$$

$$\Psi = VAR(\zeta) = \begin{bmatrix} * & & \\ * & * & \\ ? & ? & ? \end{bmatrix}$$

Model a Retest Effect




```
TITLE:    Latent Growth Curve
          with retest effect
DATA:     FILE = blah.dat ;
VARIABLE: NAMES= y1-y4 x1 ;
MODEL:
  i s | y1@0 y2@0 y3@0 y4@0 ;
  i s on x1 ;
  r by y2-y4@1 ;
  [r*] ;
  ! test what happens
  ! relaxing the constraints
  ! below...
  r@0 ;
  r with i @0 ;
  r with s @0 ;
```

LAB SESSION

(time permitting)

QUESTIONS?

END OF DAY TWO